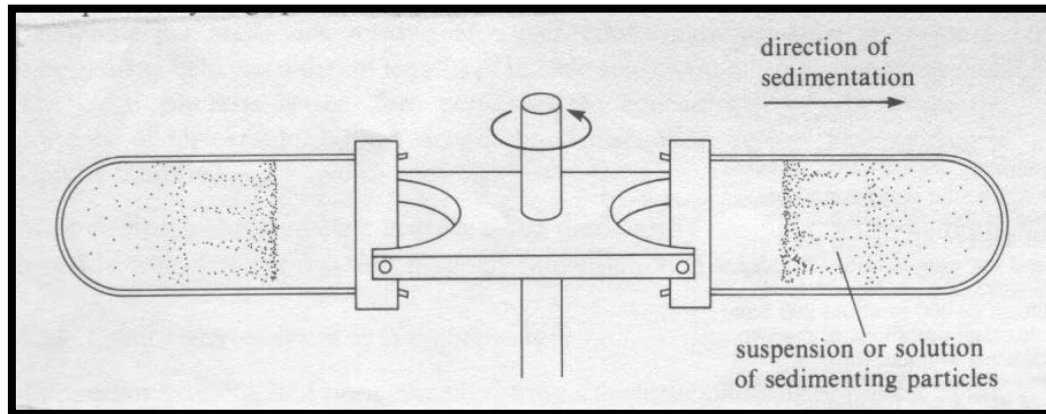


Datum/Zeit	Veranstaltungsort	Thema
Mo, 08.02.2010 10.00-11.30	Hörsaal Institut für Glaschemie Fraunhoferstrasse 6	<i>Albert Einstein and the Viscosity of Macromolecules</i>
Mo, 08.02.2010 12.15-13.45	Hörsaal Haus 1, IAAC, August-Bebel-Str. 2	<i>Light Scattering and SEC-MALLs</i>
Di, 09.02.2010 12.15-13.45	Institut für Materialwissenschaft und Werkstofftechnologie, HS 124 Löbdergraben 32	<i>Dynamic Light Scattering</i>
Mi, 10.02.2010 16.15-17.45	Hörsaal 3 Carl-Zeiss-Str. 3	<i>Analytical Ultracentrifugation I</i>
Do, 11.02.2010 14.15-15.45	Döbereiner Hörsaal	<i>Analytical Ultracentrifugation II: Interactions</i>

Lecture 4. Analytical Ultracentrifugation I: Molecular weight and conformation



Steve Harding





Analytical Ultracentrifugation Conference

Welcome!

This conference on Analytical Ultracentrifugation is dedicated to **Nobel Laureate The Svedberg (biography)** at the 125th anniversary of his birth on August 30, 1884. The Svedberg is the father of the analytical ultracentrifuge, an invention that allowed the determination of particle size distributions of colloids, the determination of molecular weights of macromolecules and the proof that macromolecules exist. The ultracentrifuge helped to put The on his way to the Nobel Prize in Chemistry in 1926 for his work on disperse systems. The pioneering work of Svedberg and his colleagues on colloids and macromolecular compounds laid the foundations for far-reaching progress in molecular biology, macromolecular chemistry and biochemistry as well as colloid science. The **envisaged topics of this special conference** try to follow The Svedberg's broad range of scientific interests related to the Analytical Ultracentrifuge.

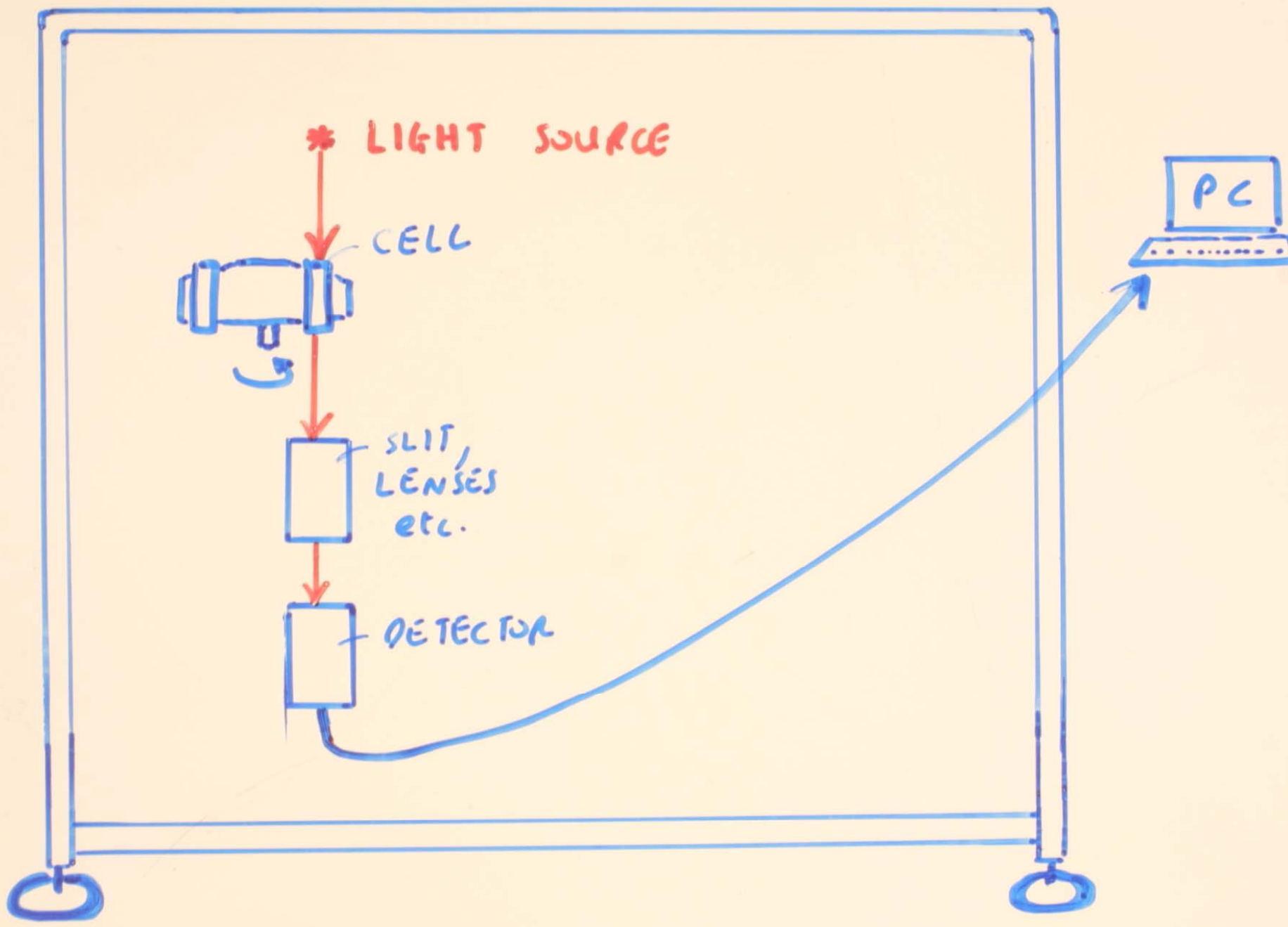


The Svedberg



Molecular weight: analytical ultracentrifugation







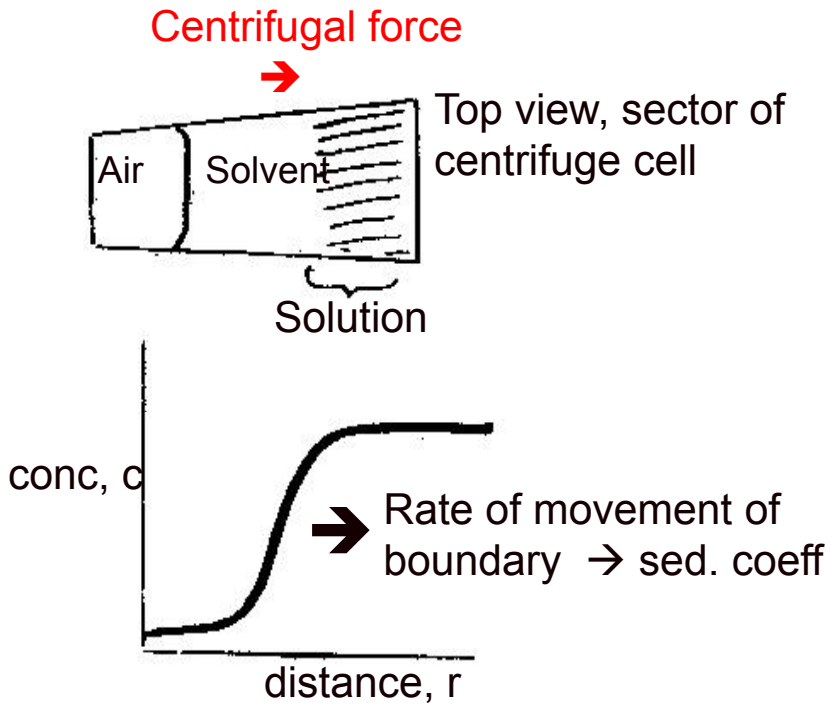


1. *Molecular weight and molecular weight distribution analysis*

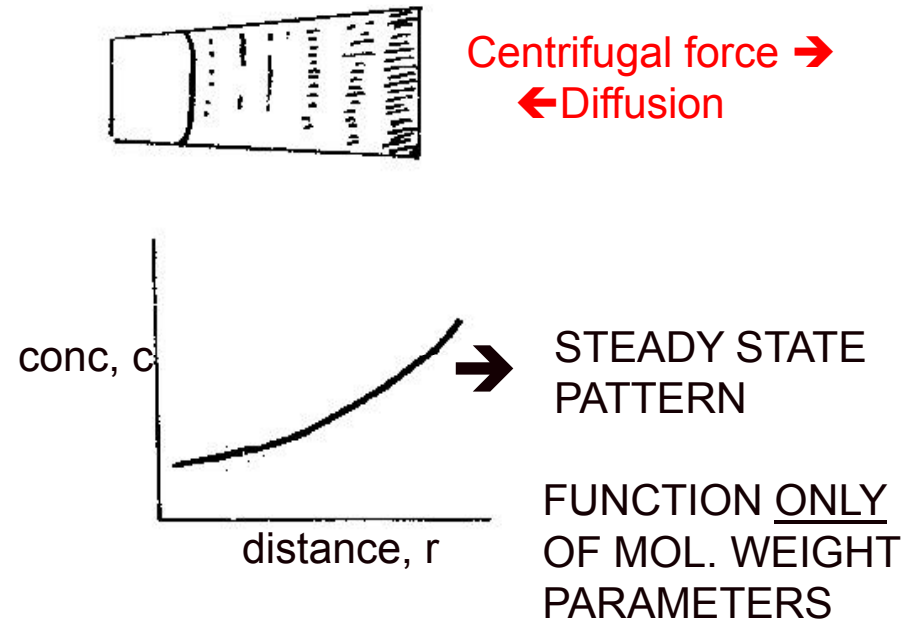
2. *Conformation and flexibility analysis*
 - *general (rods, spheres, coils etc)*
 - *polymer flexibility*
 - *protein conformation: ellipsoids and bead models*

Analytical ultracentrifugation:

Sedimentation Velocity

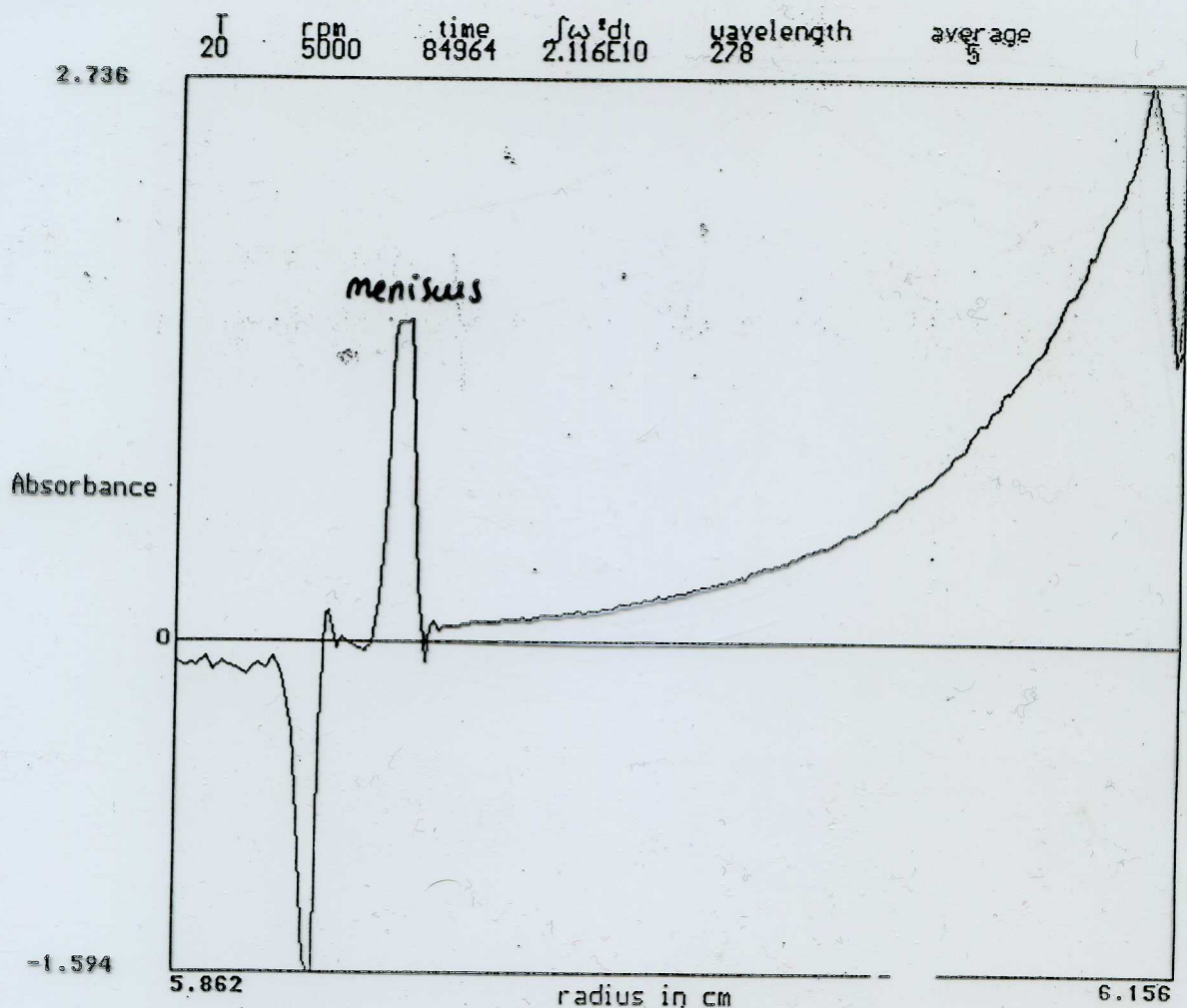


Sedimentation Equilibrium

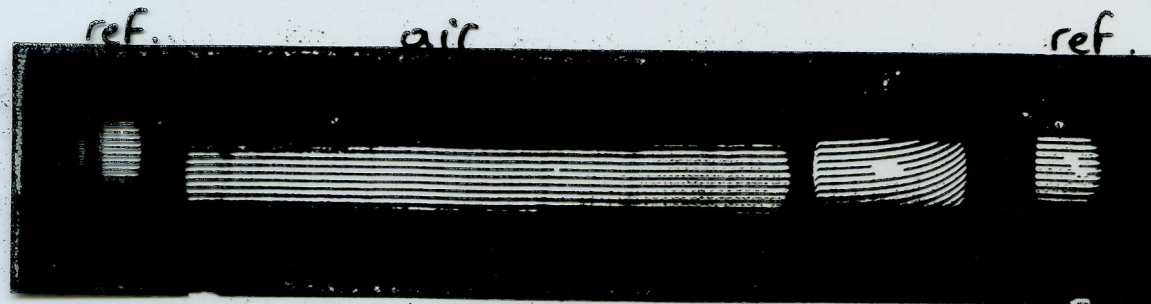


OPTICAL RECORDS FROM SEDIMENTATION EQUILIBRIUM

Absorption (I_{9M}) $M_w \sim 1.0 \times 10^6$



Interference (reduced mucin) $M_w \sim 50000$



SEDIMENTATION EQUILIBRIUM TRACE :

Interference Optics - Ovalbumin

$$M_w = 45000$$

centrifugal field



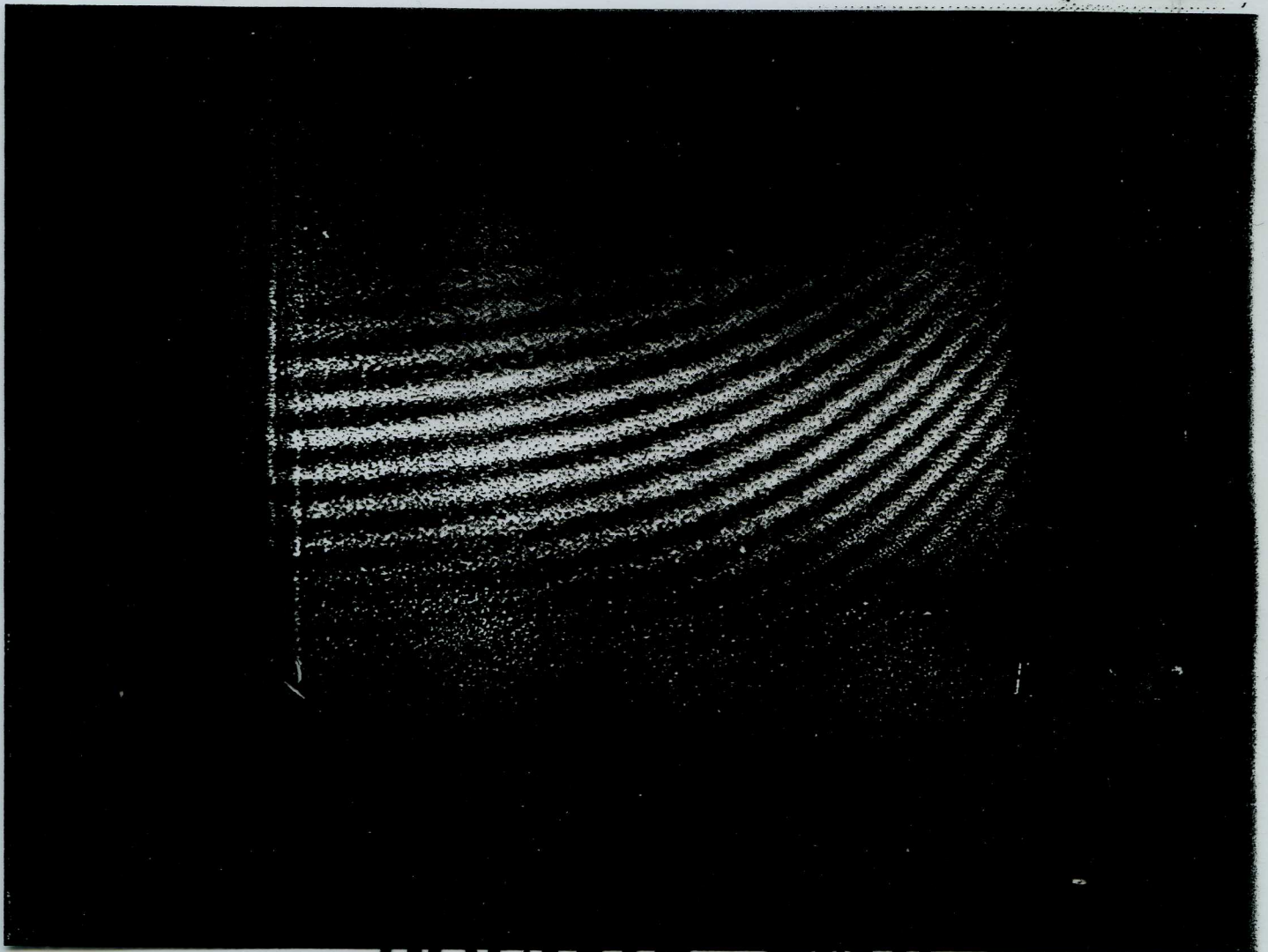
m (meniscus)



b (base cell)



cell

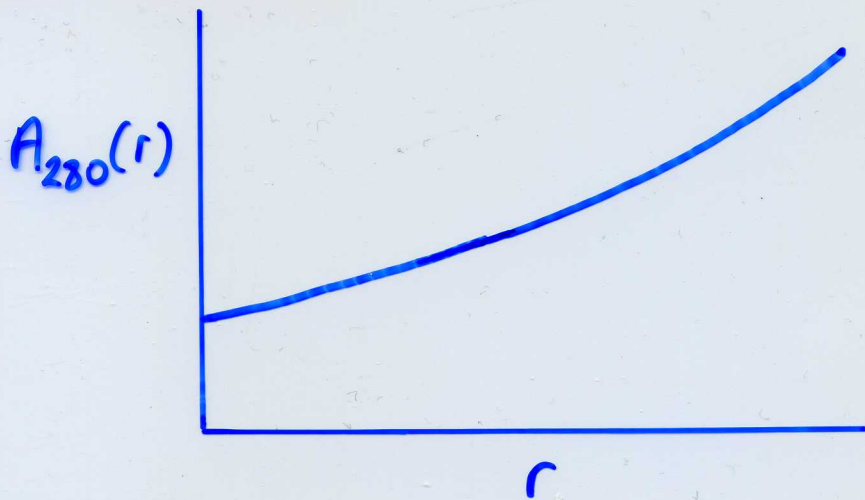


distance from

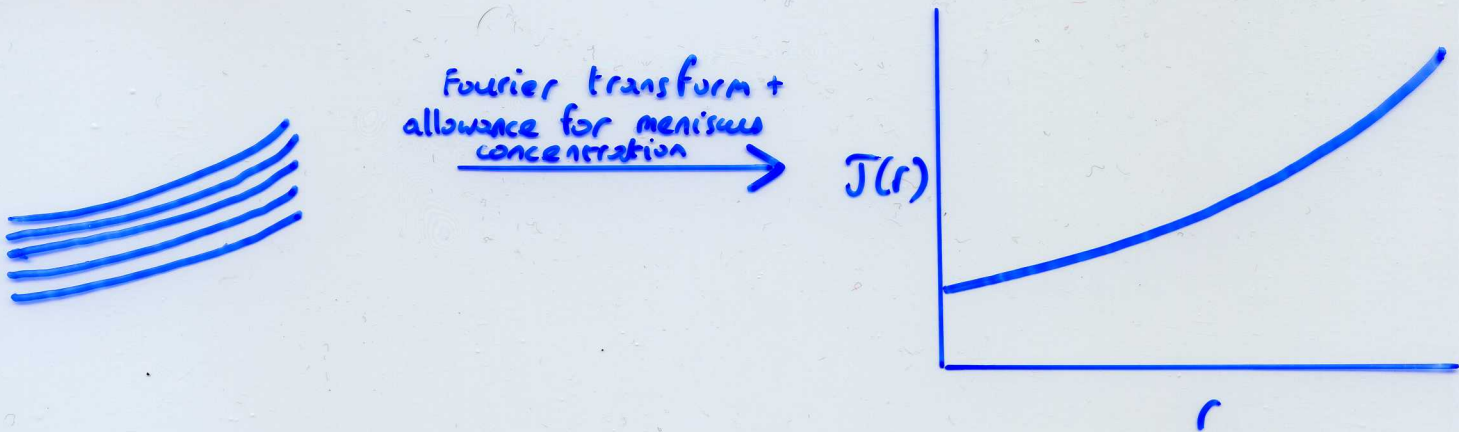
rotor centre

∴ To summarize both uv-absorption and interference optics provide a record of the concentration distribution of sedimentation equilibrium in the ultracentrifuge cell

① UV ABSORPTION

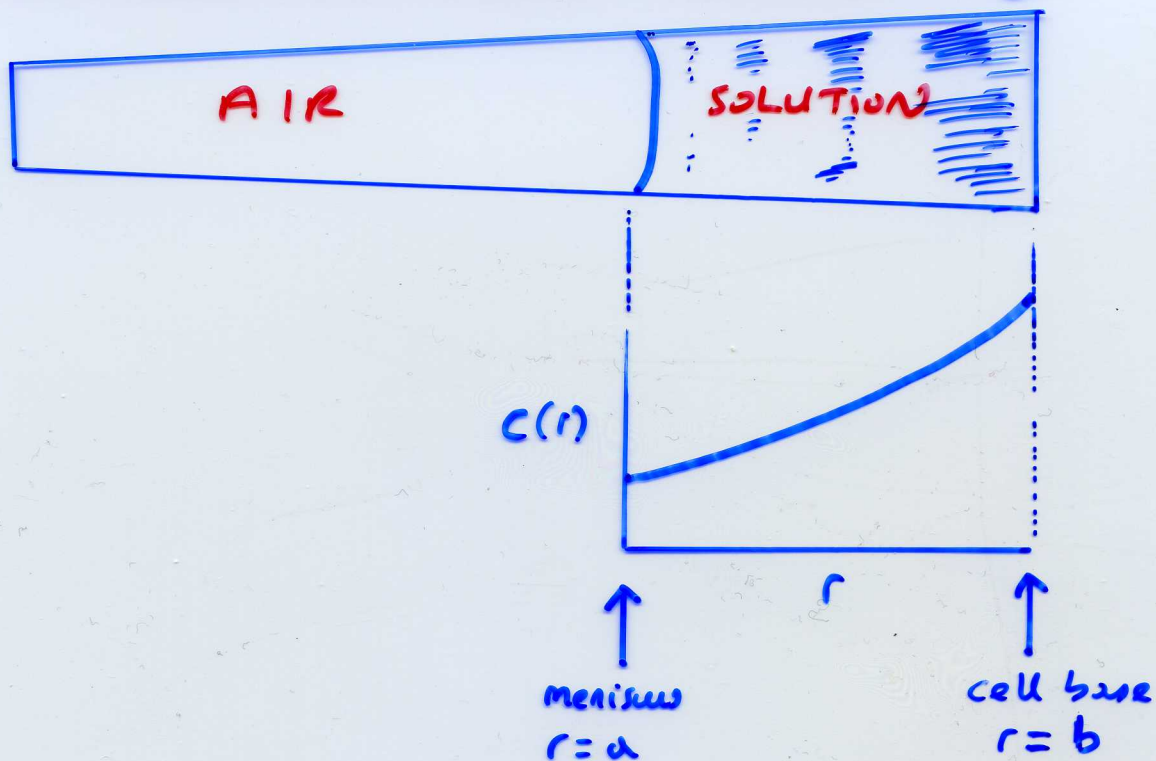


② INTERFERENCE



IN BOTH CASES } $A(r) \propto c(r)$
THE RECORD IS A DIRECT
RECORD OF CONCENTRATION } $J(r) \propto c(r)$

DISTANCE FROM THE ROTOR CENTRE, r



In working out molecular weights we can either use concentrations in the centrifuge cell, $c(r)$, directly (g/ml) or use absorbances $A(r)$ or fringe displacements $J(r)$

At sedimentation Equilibrium sedimentation & diffusion forces are = and opposite

$$\omega^2 r M (1 - \bar{v} \rho) \quad = \quad \frac{RT}{c(r)} \frac{dc(r)}{dr}$$

(from sedimentation) (from diffusion)

$$\therefore \frac{1}{c(r)} \frac{dc(r)}{dr} = \frac{\omega^2 r M (1 - \bar{v} \rho)}{RT}$$

N.B. For heterogeneous / non-ideal systems M should be $M_{w,app}$

THE IMPORTANT EQUATIONS FOR SEDIMENTATION EPM.

Fundamental equation :
$$\frac{1}{c(r)} \frac{dc(r)}{dr} = \frac{\omega^2 r M_{w,app} (1-\bar{v}\rho)}{RT}$$

+ its integrated form :
$$c(r) = c(a) e^{\omega^2 M_{w,app} (1-\bar{v}\rho) (r^2 - a^2) / 2RT}$$

These can be manipulated in a number of ways

① Logarithmic form over the whole distribution (from $r=a$ to $r=b$)

$$M_{w,app} = \left[\ln \frac{c(b)}{c(a)} \right] \cdot \frac{2RT}{\omega^2 (1-\bar{v}\rho) (b^2 - a^2)}$$

(n.b. at low concentration $M_{w,app} \approx M_w$)

② Integral form over the whole distribution

$$M_{w,app} = \left[\frac{c(b) - c(a)}{c^0} \right] \frac{2RT}{\omega^2 (1-\bar{v}\rho) (b^2 - a^2)}$$

c^0 : initial concentration loaded into the centrifuge cell

③ "Local" or point average molecular weights

$$M_{w,app}(r) = \frac{d \ln c(r)}{dr^2} \cdot \frac{2RT}{(1-\bar{v}\rho) \omega^2}$$

④ M^* form

$$M^*(r) = \frac{c(r) - c(a)}{\left[\frac{(1-\bar{v}_p)\omega^2}{2RT} \right] \cdot \left[c(a)(r^2 - a^2) + 2 \int_a^r \{c(r) - c(a)\} dr \right]}$$

and $M_{w,app} = M^*(r \rightarrow b)$

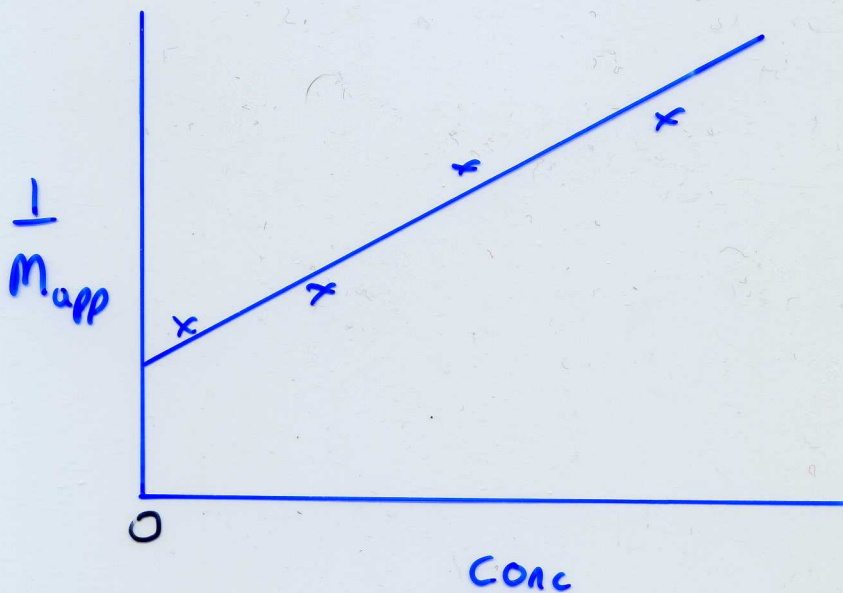
THERMODYNAMIC NON-IDEALITY

Values for M so obtained (at a finite concentration c) are only apparent values (M_{app}) because of non-ideality [although in practise, for proteins

$M_{app} \approx M$ especially for concentrations $\leq 1.0 \text{ mg/ml}$]

for glycoconjugates + polysaccharides, an extrapolation

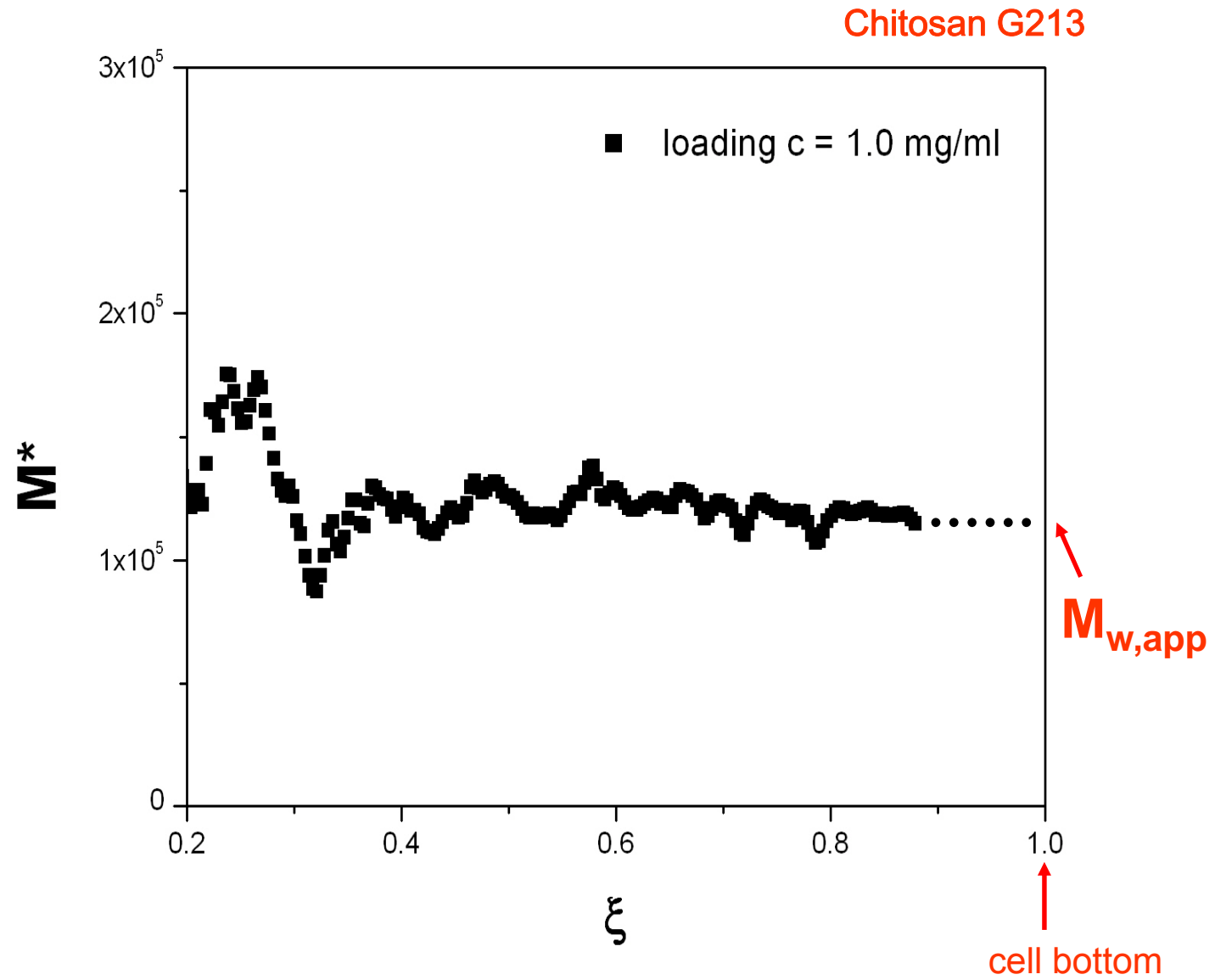
MAY be necessary



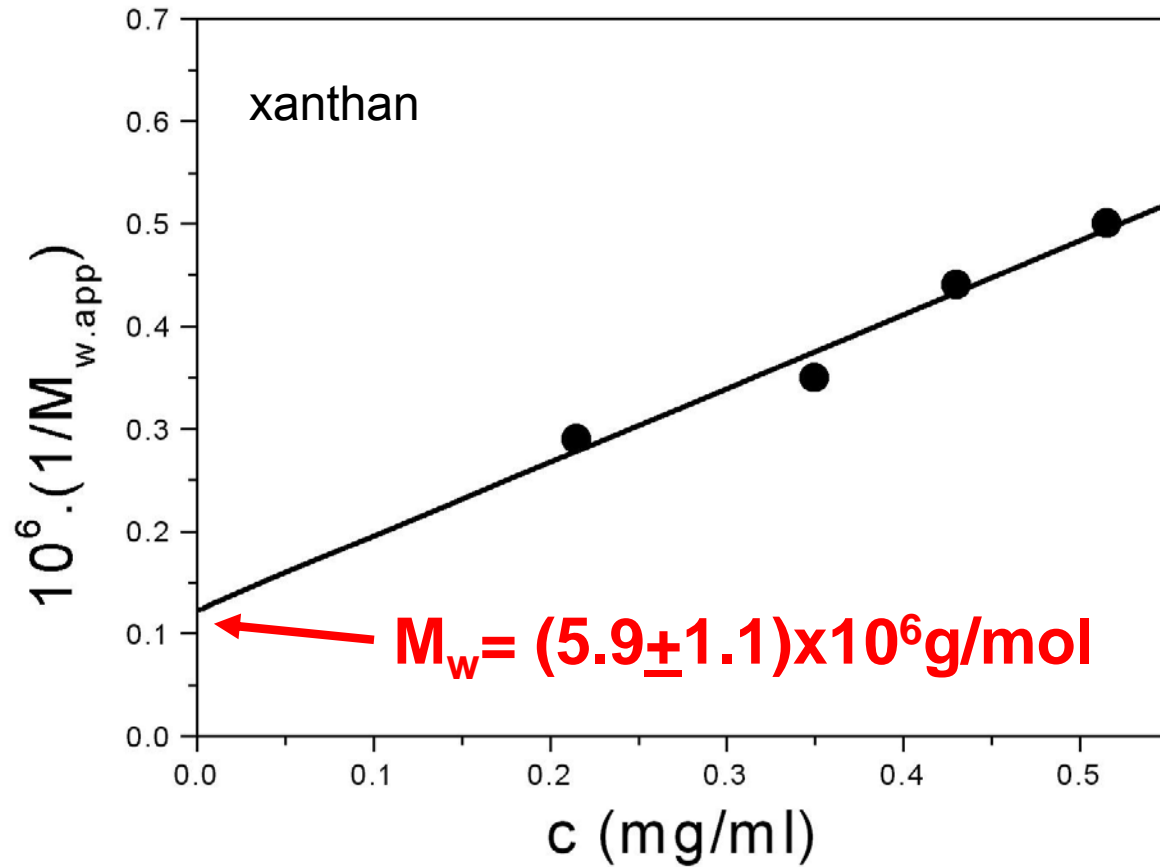
$$\frac{1}{M_{app}} = \frac{1}{M} (1 + 2BMc)$$

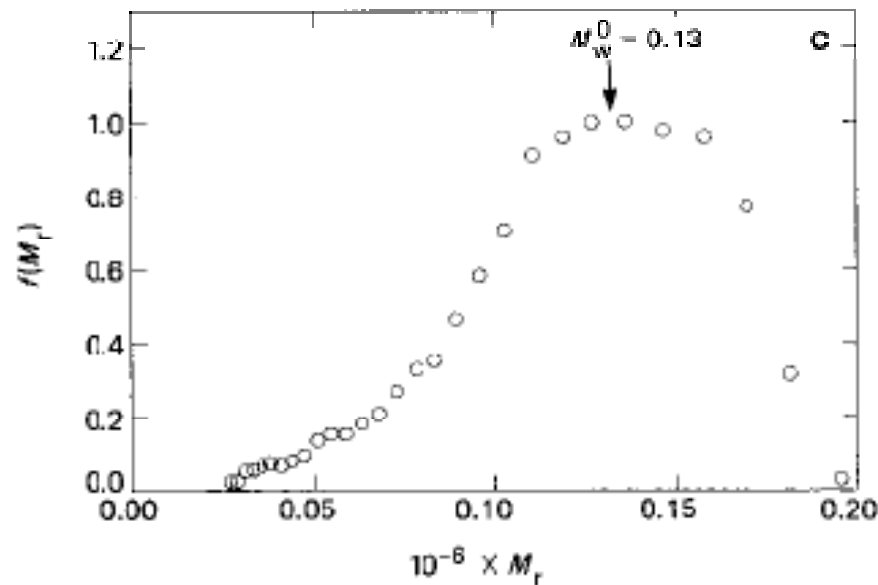
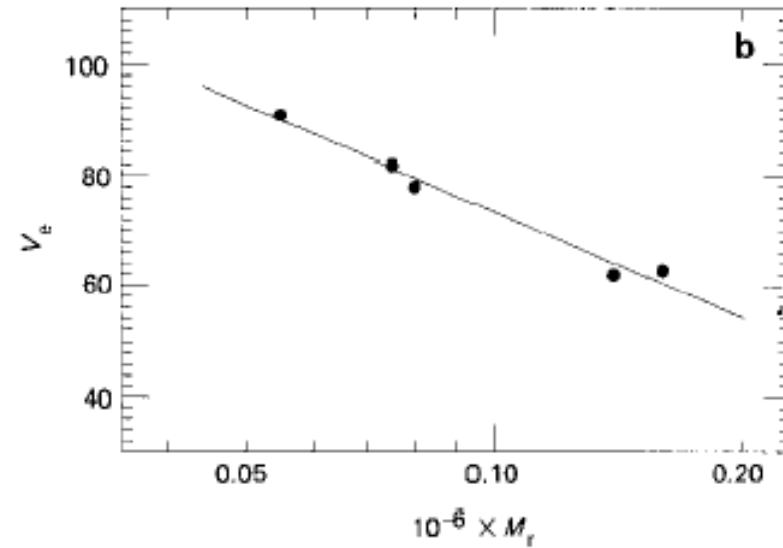
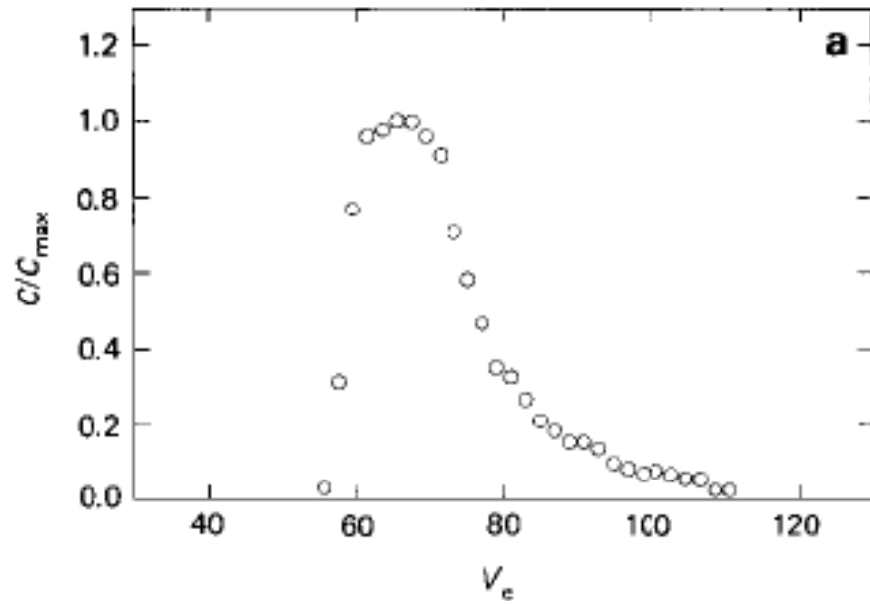
2nd virial coefficient

Extraction of $M_{w,app}$ from sedimentation equilibrium and "MSTAR" analysis



Extraction of $M_{w,app}$ from sedimentation equilibrium and "MSTAR" analysis

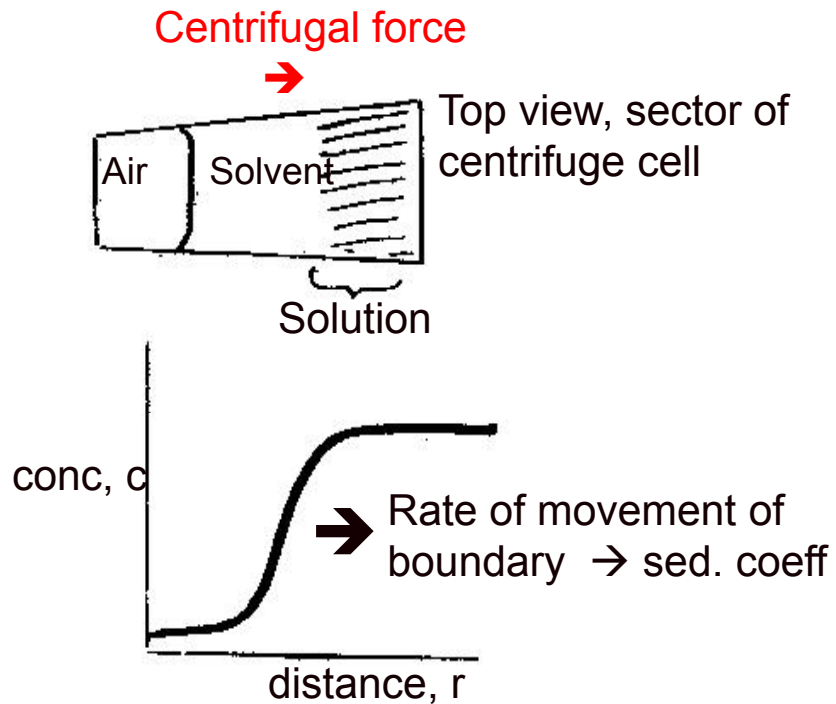




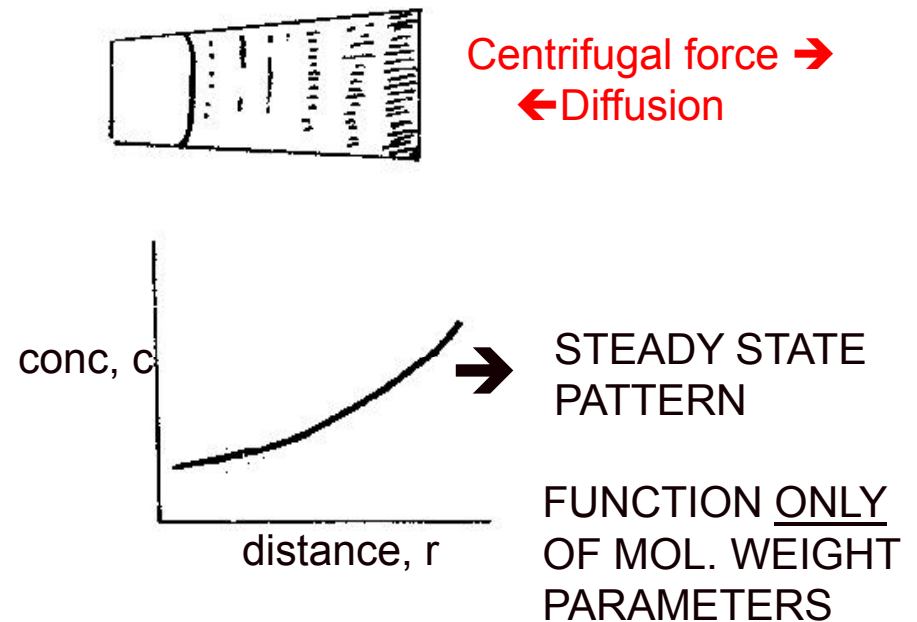
SEC - sedimentation equilibrium
mol. wt distribution: alginate

Ball A, Harding SE & Mitchell J,
Int. J. Biol. Macromol., 1988

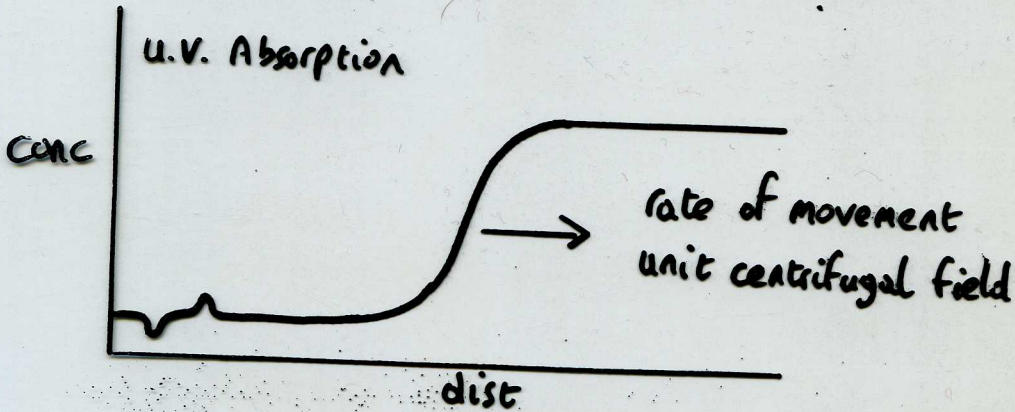
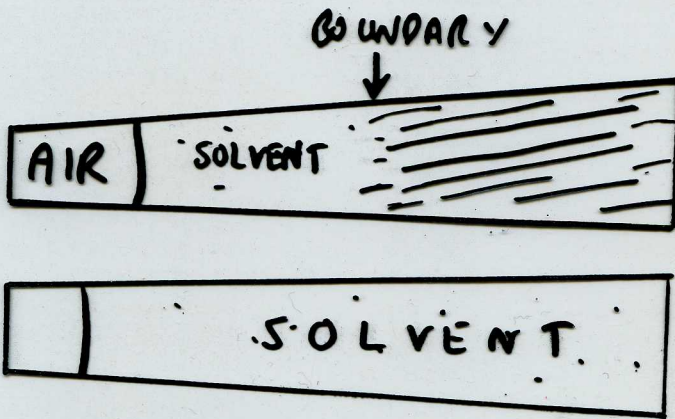
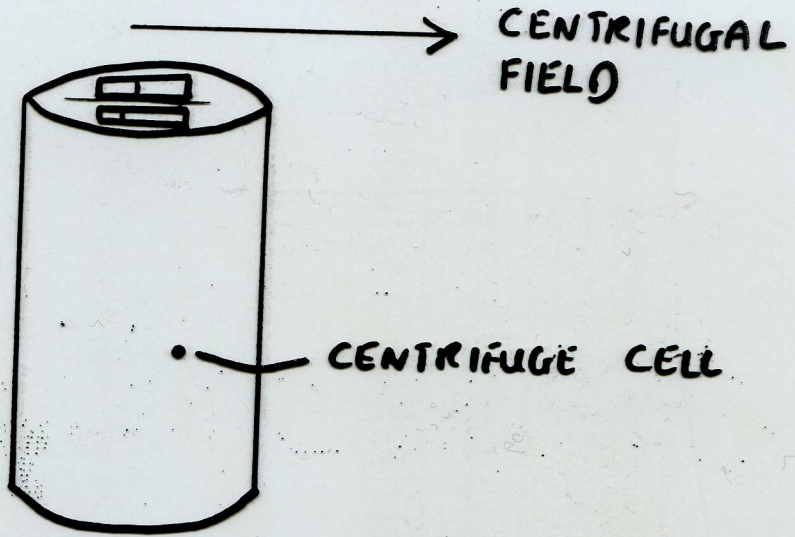
Sedimentation Velocity



Sedimentation Equilibrium

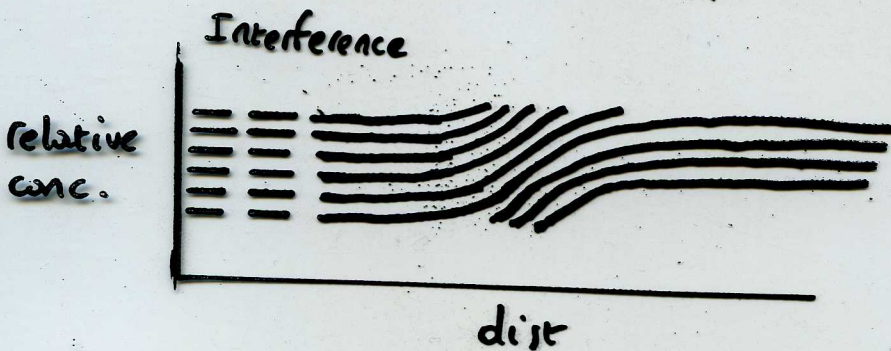


SEDIMENTATION VELOCITY



SEDIMENTATION
COEFFICIENT

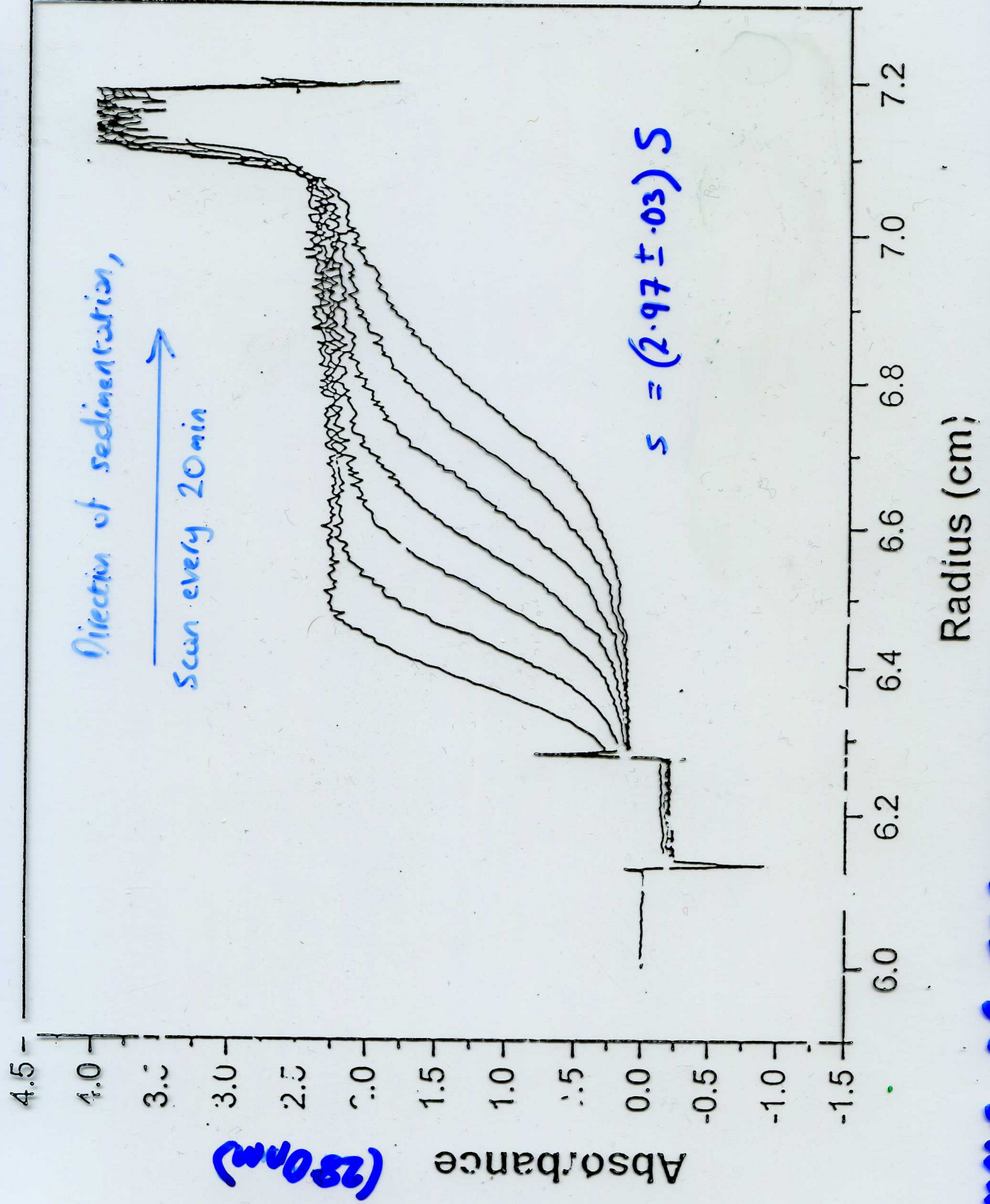
$$S_{20,W}^0$$



UNIT: SVEDBERGS

$$1 S = 10^{-13} \text{ sec}$$

β -LACTOGLOBULIN , 25.0°C, 50000 rpm



EXAMPLE OF SEDIMENTATION VELOCITY USING ABSORPTION OPTICS

Interference Optics:

Tomato Beshy Stunt Virus

11000 rpm

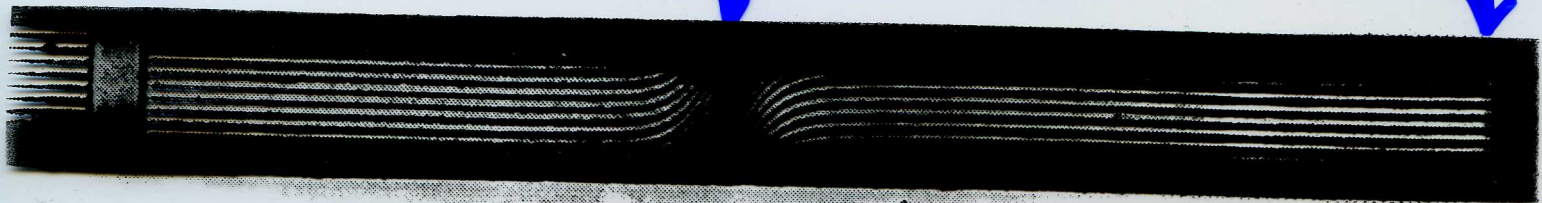
20.0°C

$s = (130 \pm 2) S$

Microisens
↓

Centre of
Sedimenting
boundary
↓

bottom of
cell
↓



Direction of
Sedimentation

EXAMPLE OF SEDIMENTATION VELOCITY
USING REFRACTOMETRIC (INTERFERENCE) OPTICS

Sedimentation velocity

sedimentation coefficient, s

$$= \frac{\text{velocity}}{\text{centrifugal field}}$$

$$= \frac{v}{\omega^2 r}$$

ω = angular velocity of rotor (radians/sec)

r = distance of particle from centre of rotor

Measured from rate of movement of boundary with time

s values often corrected to standard conditions for comparison purposes (20°C in water)

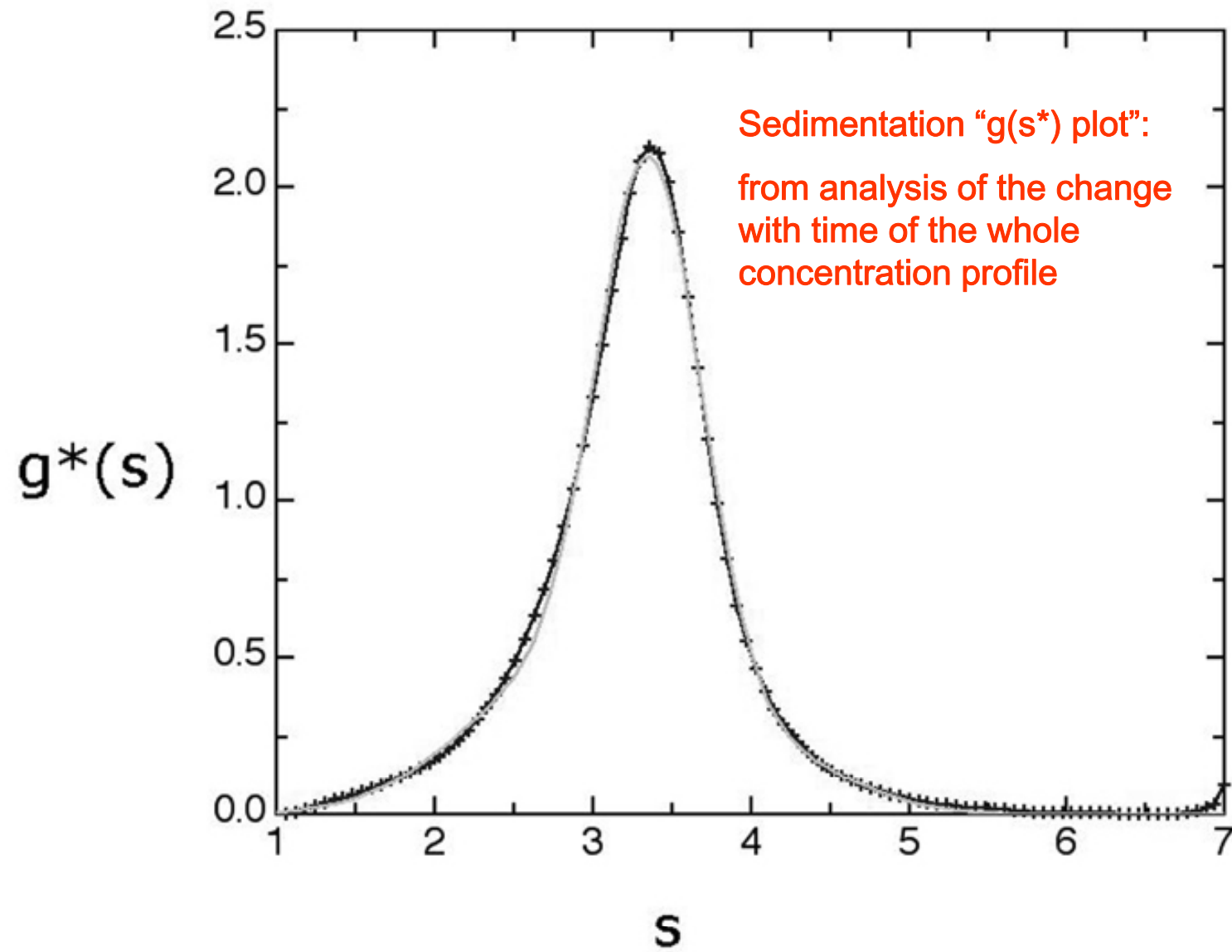
Lysozyme : $s = 1.91 \times 10^{-13}$ sec

Bovine serum albumin, $s = 5.01 \times 10^{-13}$ sec

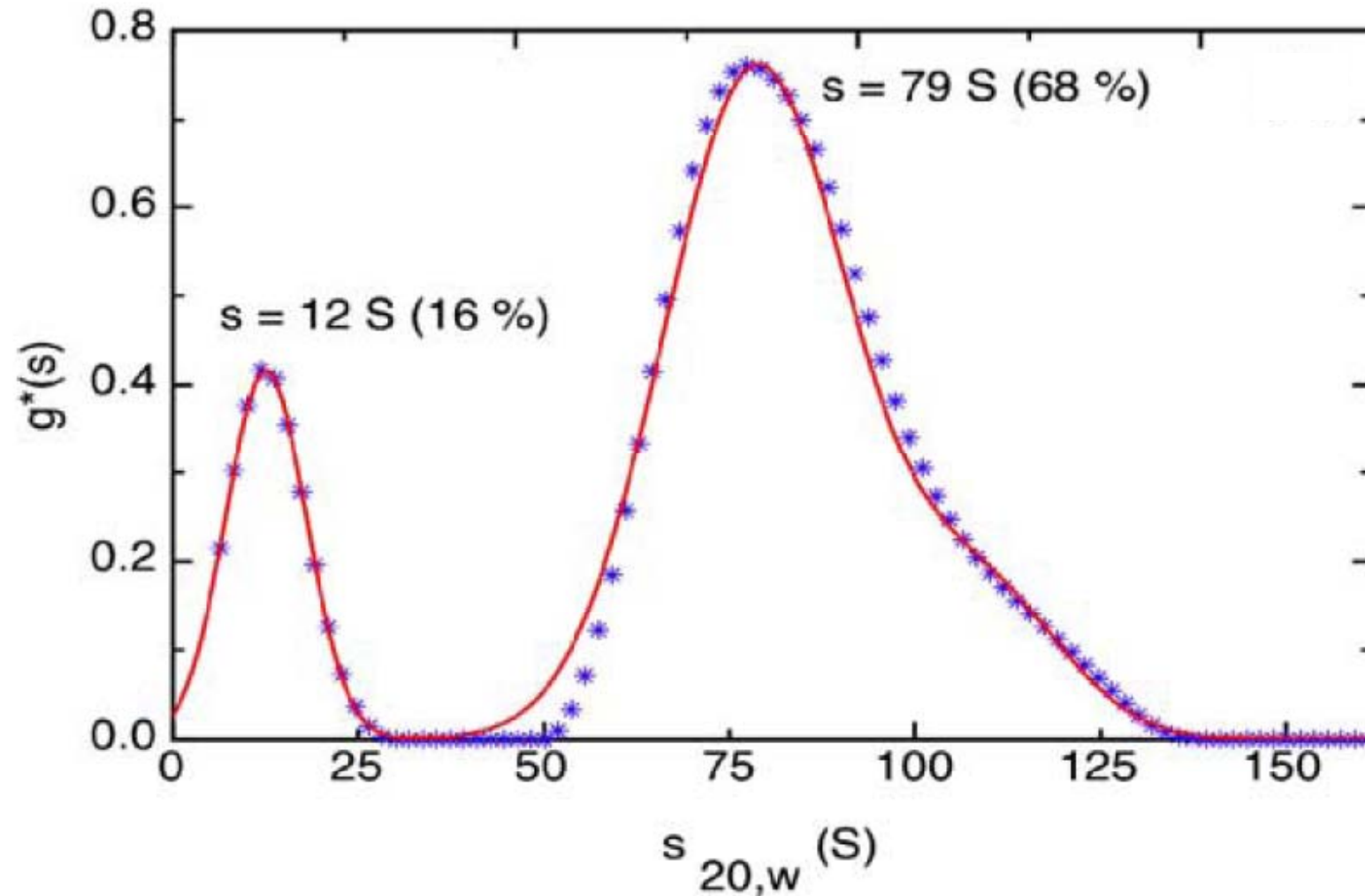
Fibrinogen $s = 7.9 \times 10^{-13}$ sec

common unit: The 'Svedberg' $S = 10^{-13}$ sec

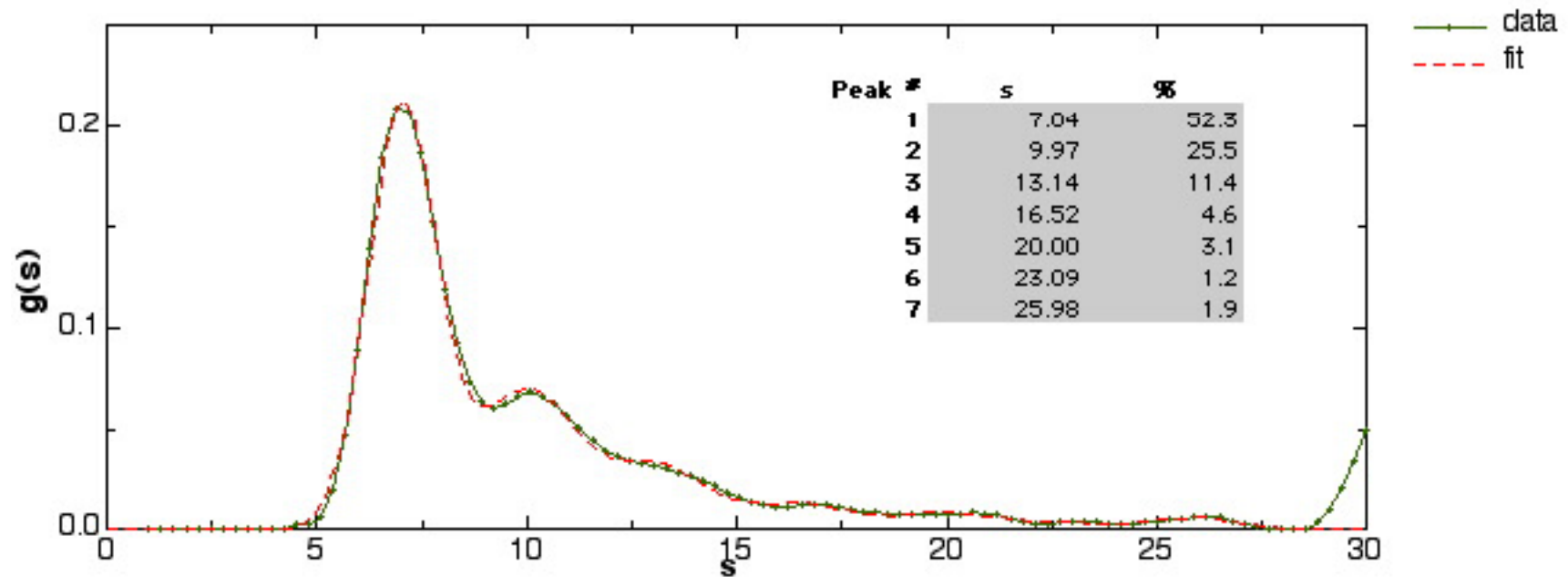
Guar, 0.75 mg/ml



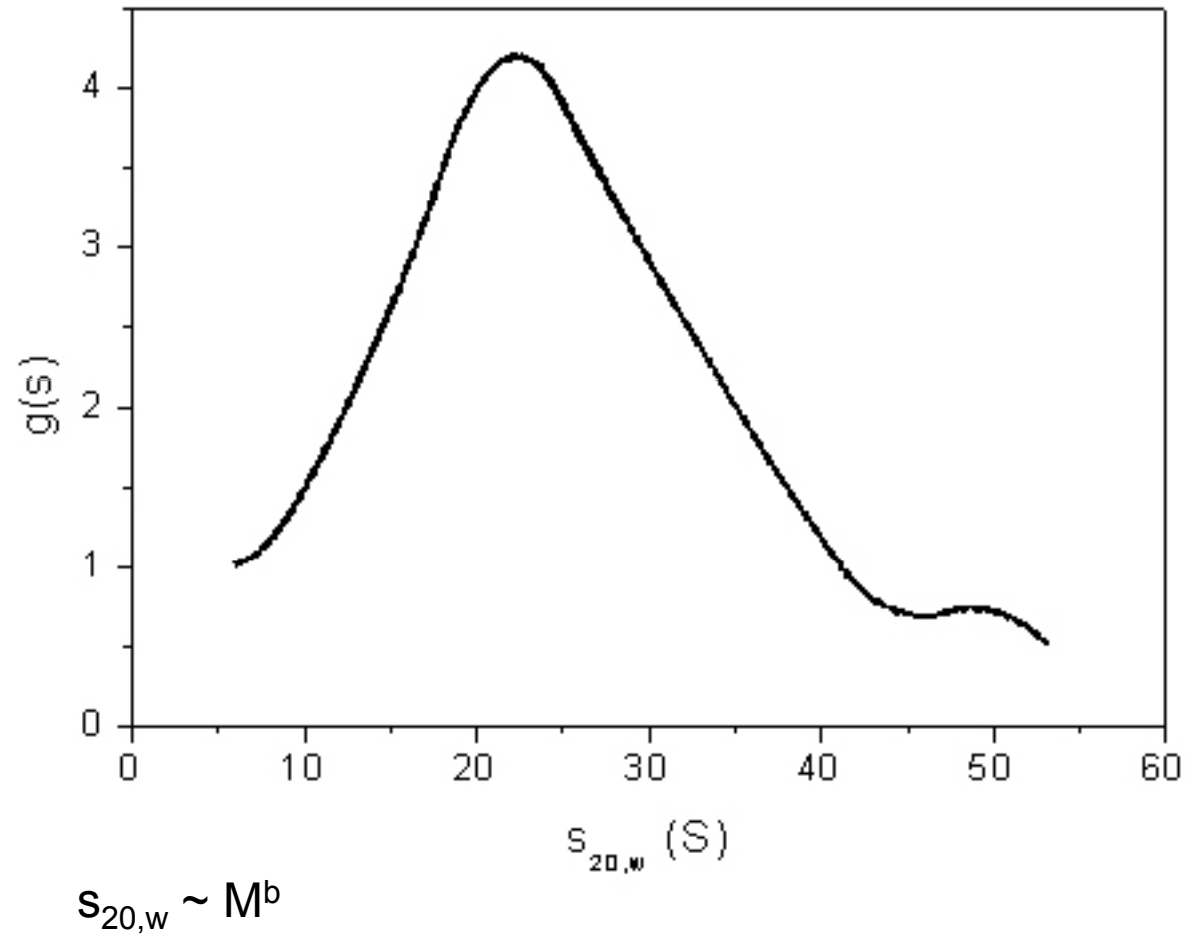
Sedimentation velocity $g^*(s)$ plot: starch



Multi-Gaussian fit estimates *proportions* of each species too:



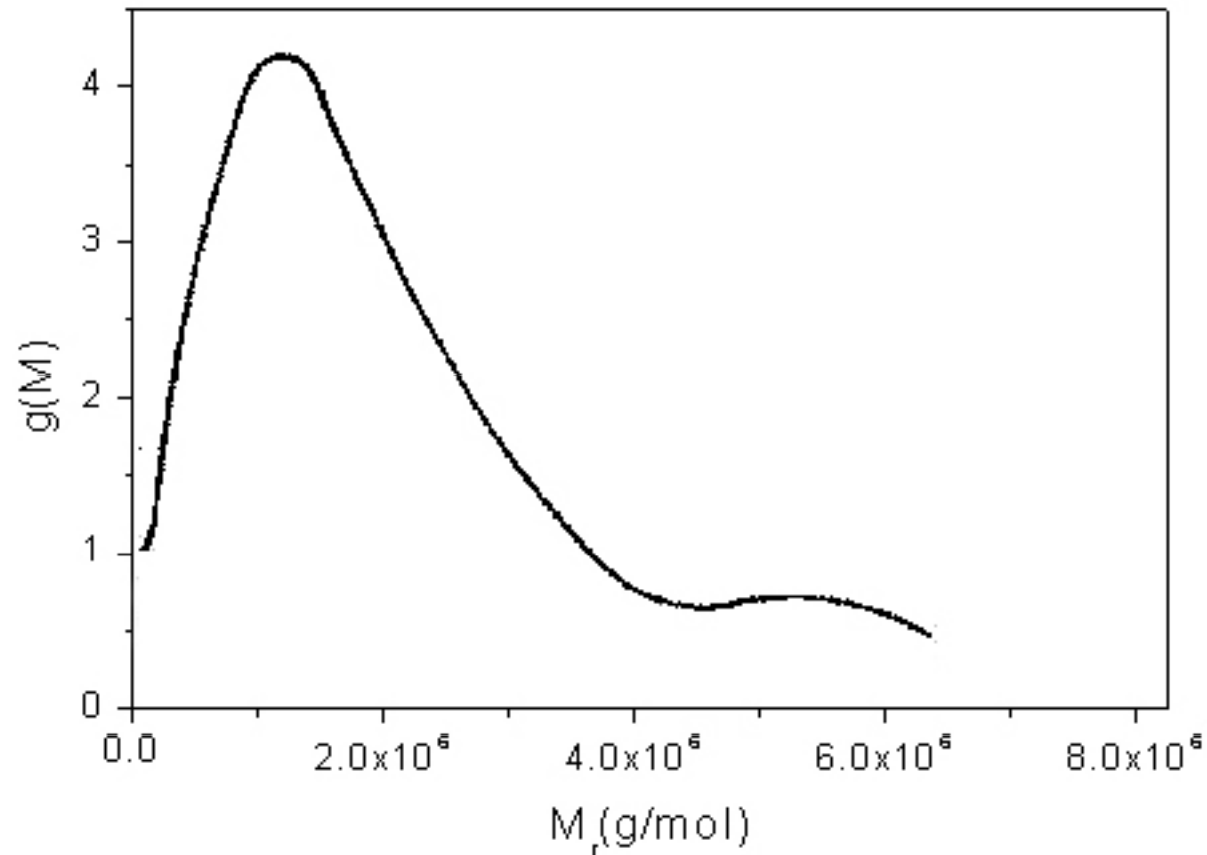
Converting a sedimentation coefficient distribution to a molecular weight distribution



Harding, S. *Adv. Carb. Chem. Biochem.*, 1989

Converting a sedimentation coefficient distribution to a molecular weight distribution

Molecular weight distribution – no column or membrane needed

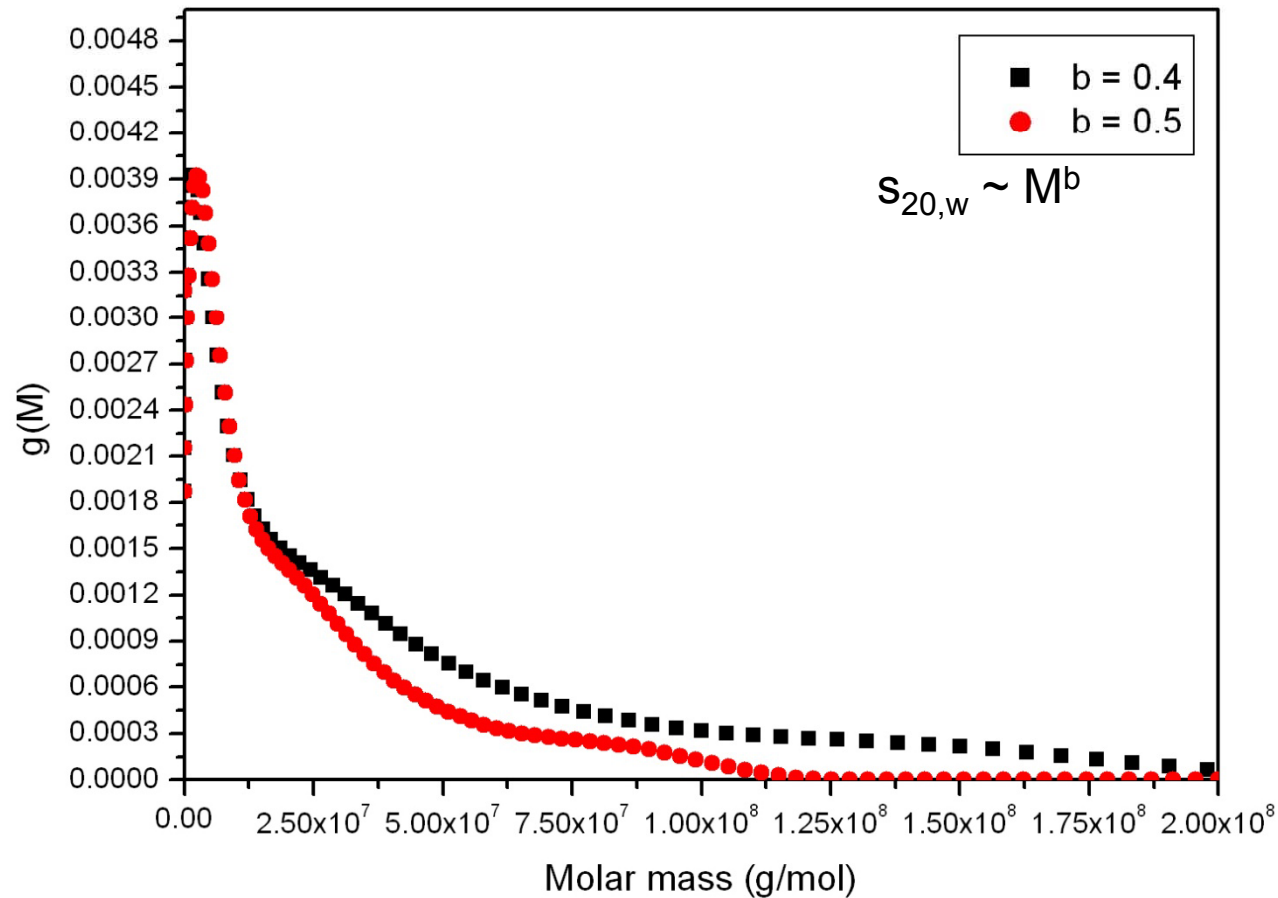


$$s_{20,w} \sim M^b \quad (b=0.5)$$

Harding, S. *Adv. Carb. Chem. Biochem.*, 1989

Converting a sedimentation coefficient distribution to a molecular weight distribution

Glycoconjugate vaccine – too large for SEC-MALLs analysis



Harding, S., Abdelhameed, A., Morris, G. (2010)

1. *Molecular weight and molecular weight distribution analysis*

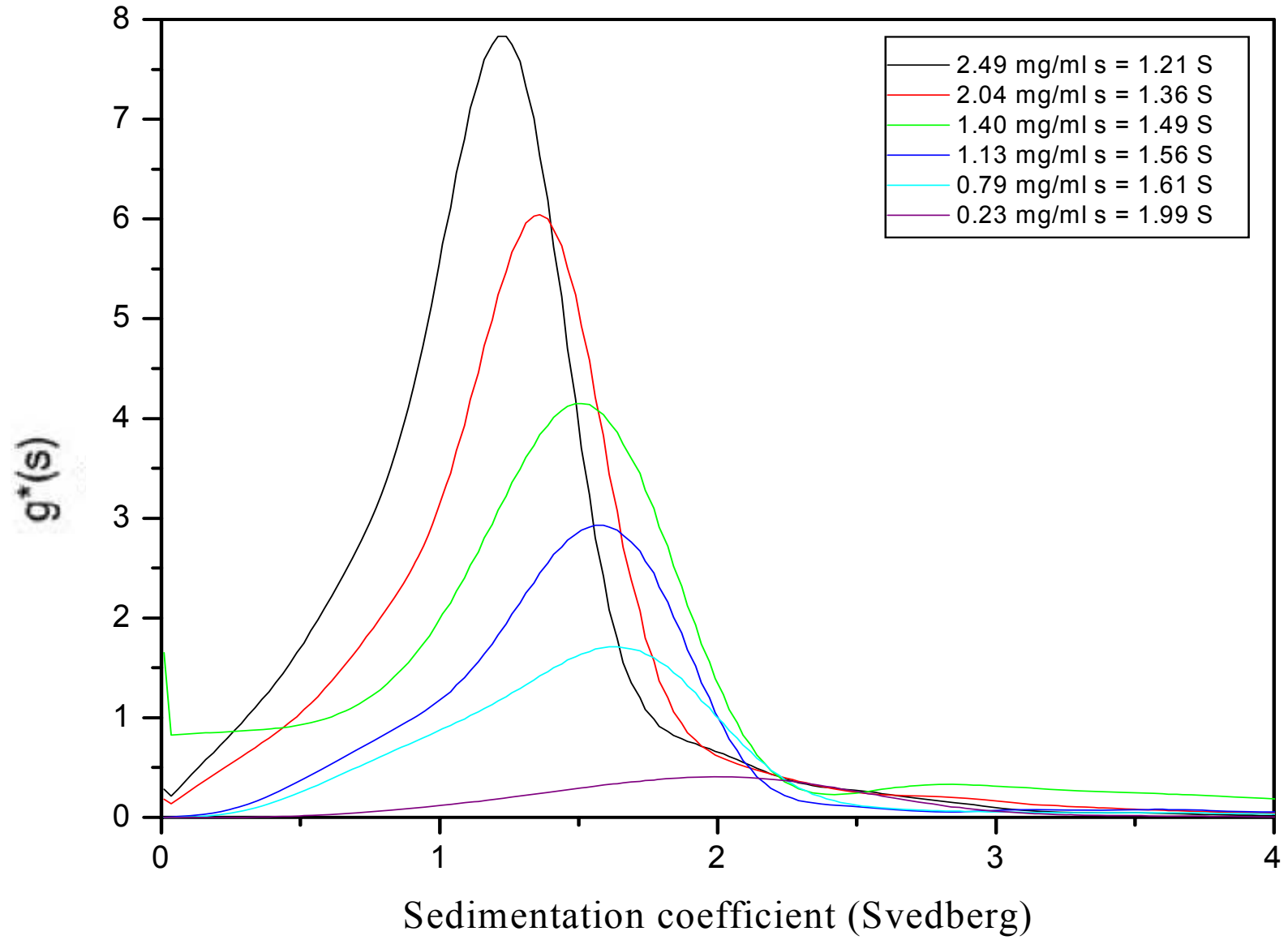
2. *Conformation and flexibility analysis*

- *general (rods, spheres, coils etc)*

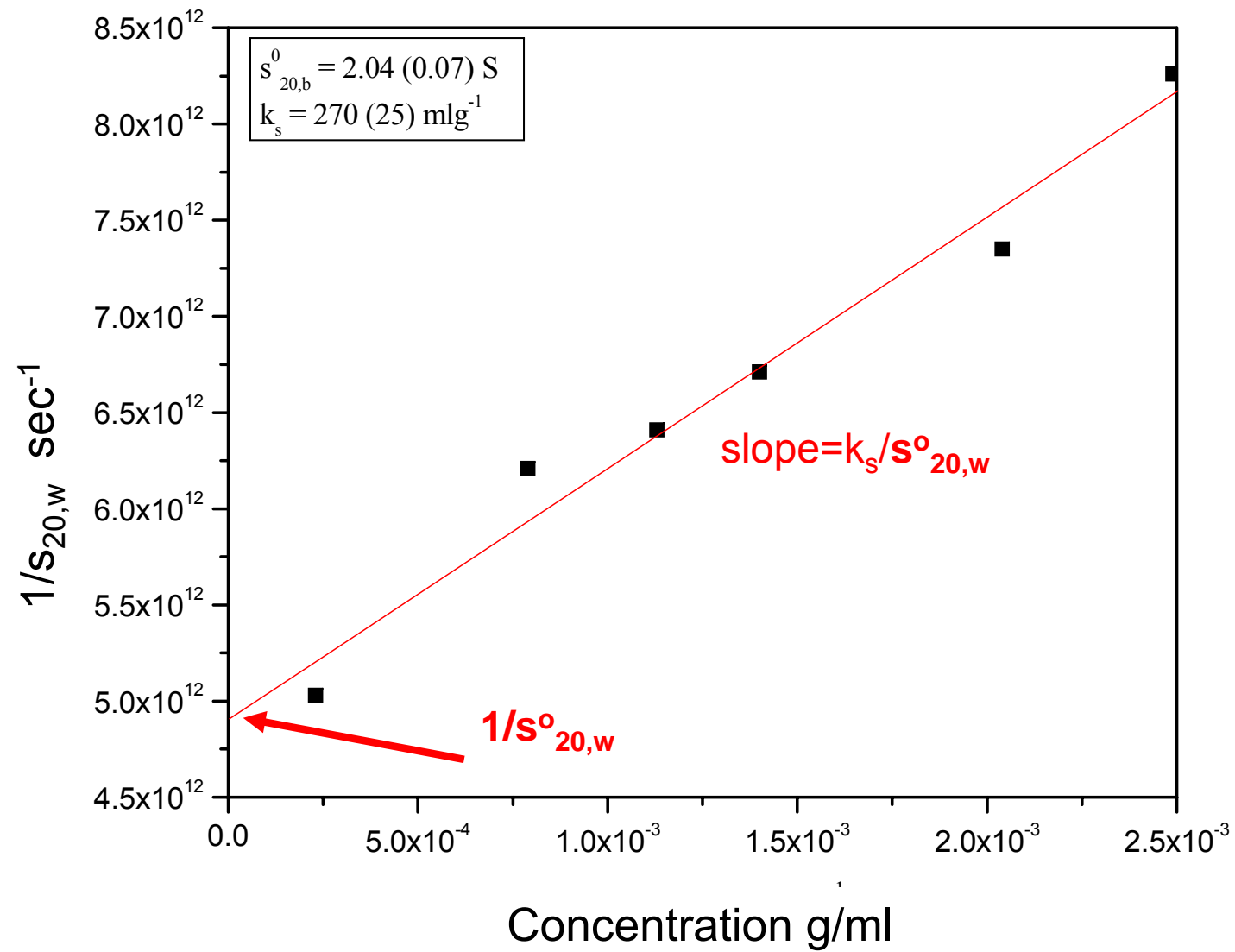
- *polymer flexibility*

- *protein conformation: ellipsoids and bead models*

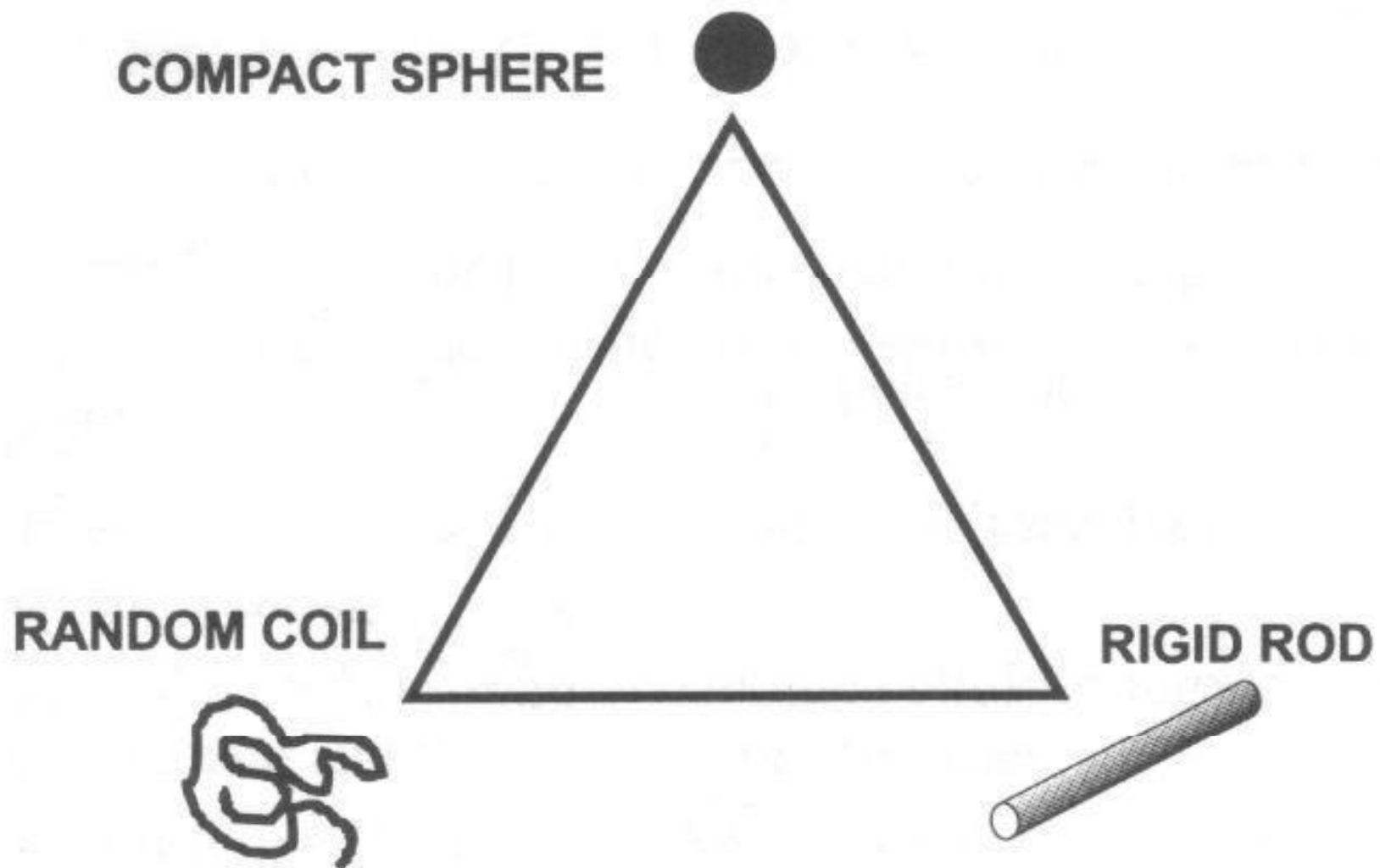
Citrus pectin

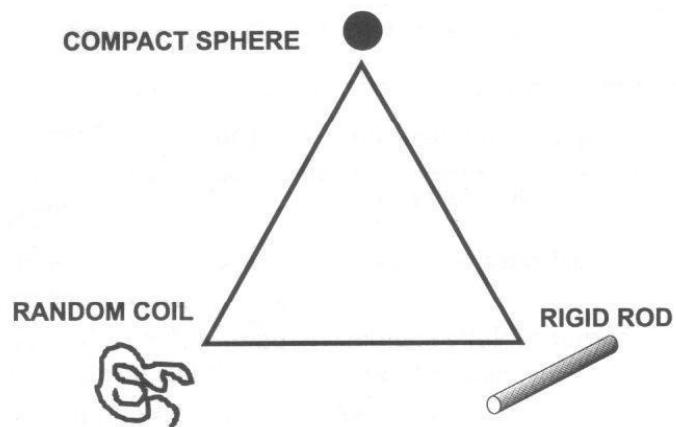


$s^{\circ}_{20,w}$ and k_s extraction



General conformation analysis: the Haug Triangle



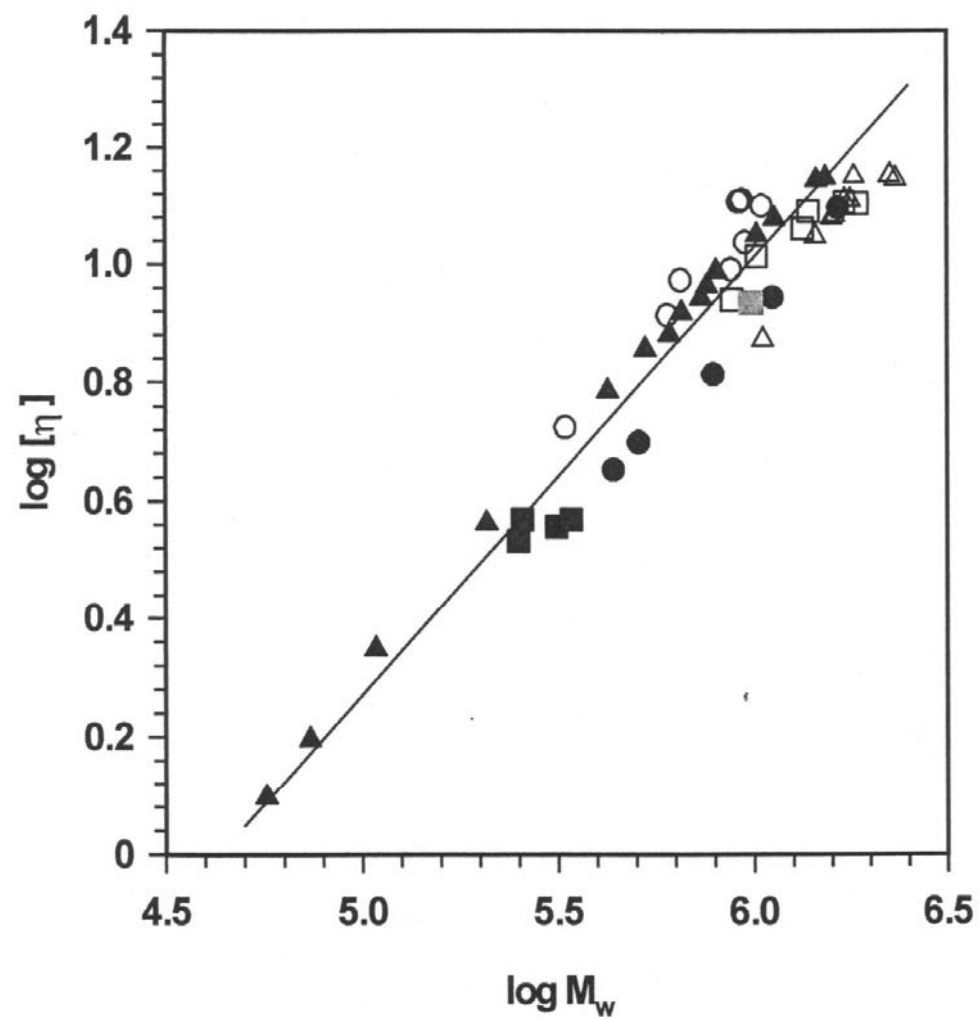


Power law, “Scaling” or “MHKS” relations:

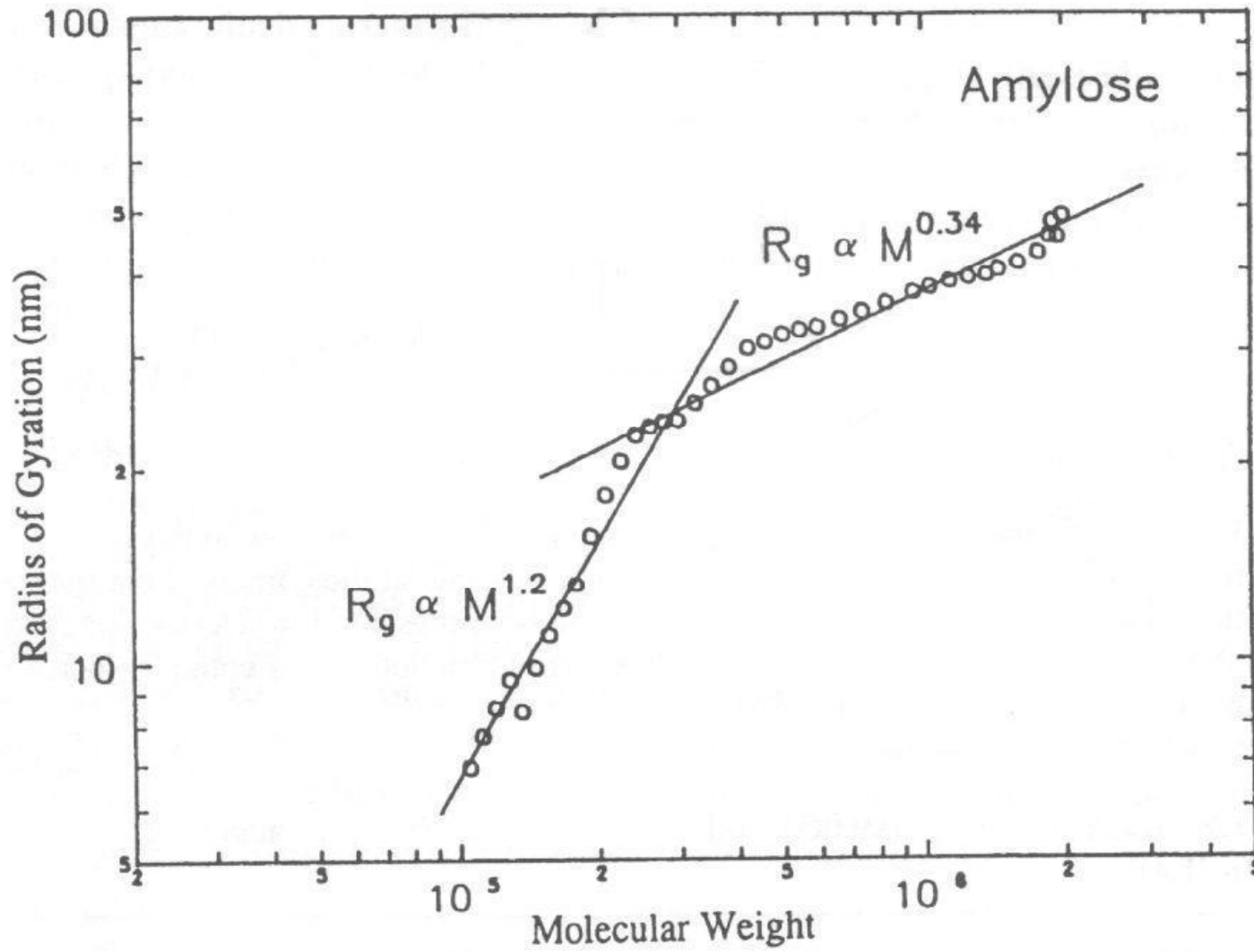
Sphere	Rod	Coil
$[\eta] \sim M^0$	$[\eta] \sim M^{1.8}$	$[\eta] \sim M^{0.5-0.8}$
$S_{20,w}^0 \sim M^{0.67}$	$S_{20,w}^0 \sim M^{0.15}$	$S_{20,w}^0 \sim M^{0.4-0.5}$
$R_g \sim M^{0.33}$	$R_g \sim M^{1.0}$	$R_g \sim M^{0.5-0.6}$

Mark-Houwink-Kuhn-Sakurada Power law plot

Galactomannans
 $a=0.74_{-0.01}$



Change in Conformation



Conformation Zoning:

Zone A: Extra-rigid rod:
schizophyllan



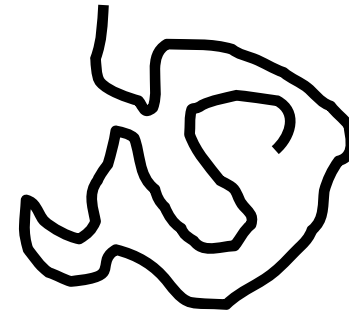
Zone B: Rigid Rod:
xanthan



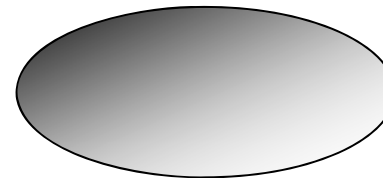
Zone C: Semi-flexible coil:
pectin



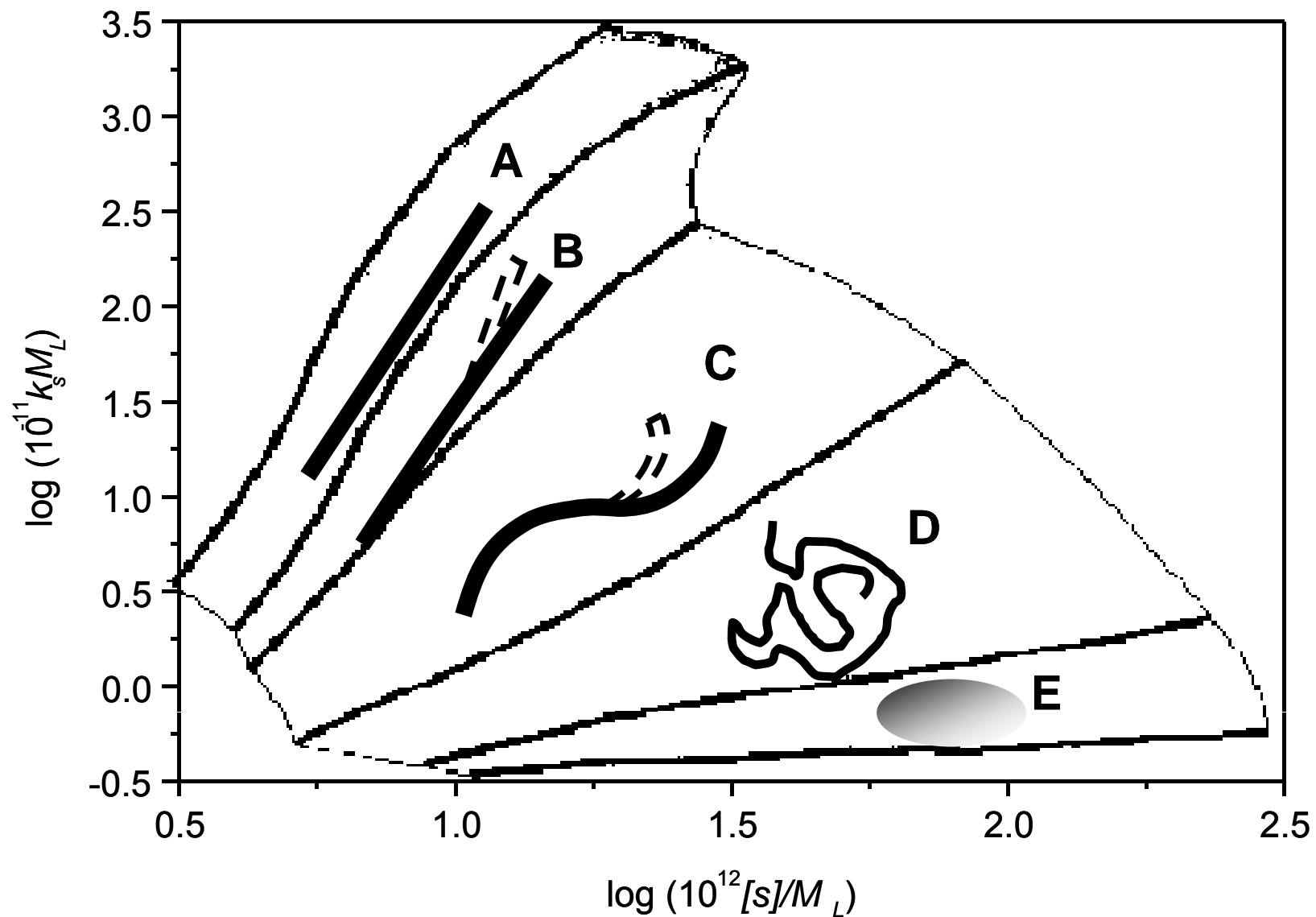
Zone D: Random coil:
dextran, pullulan



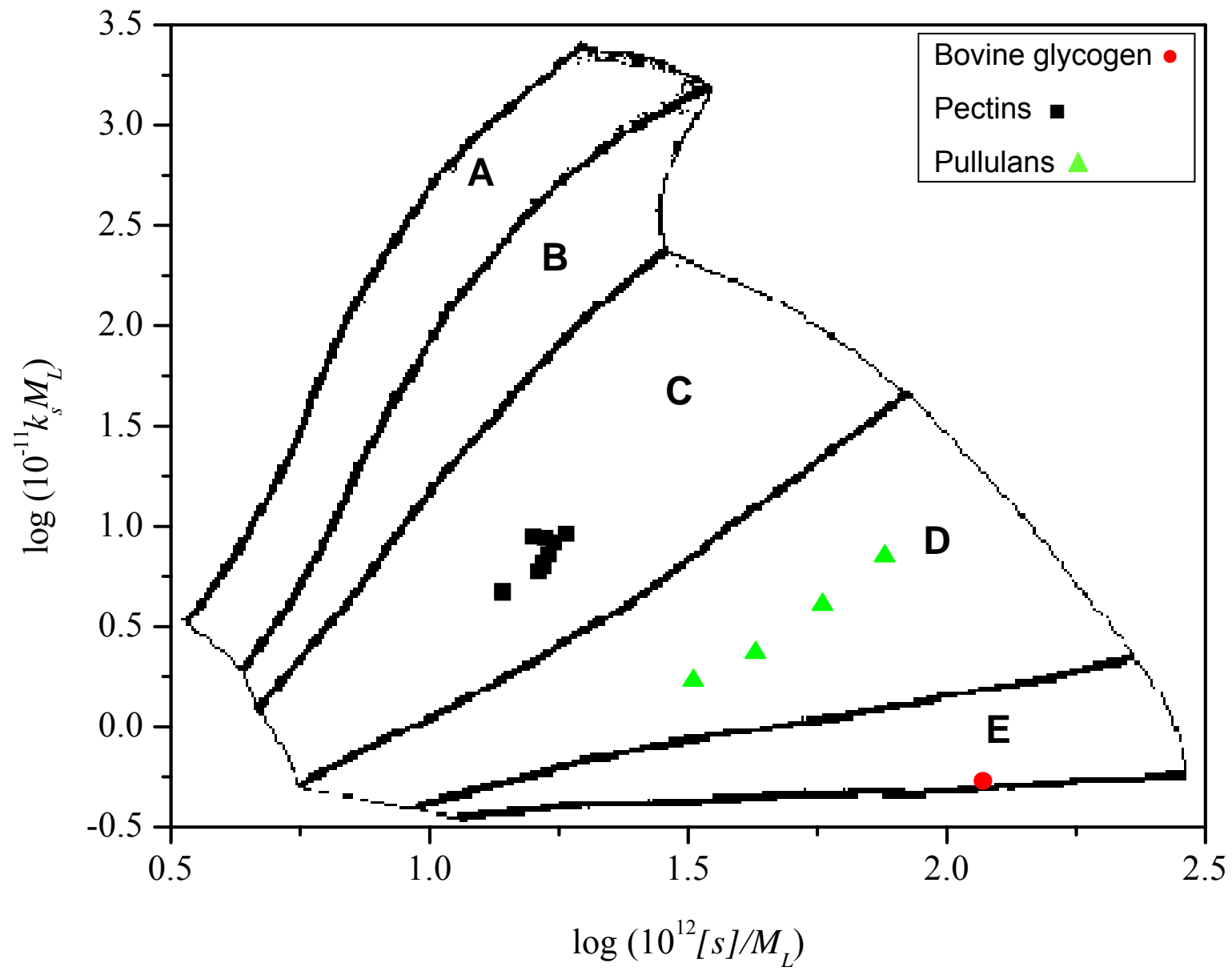
Zone E: Highly branched:
amylopectin, glycogen



Conformation Zoning:

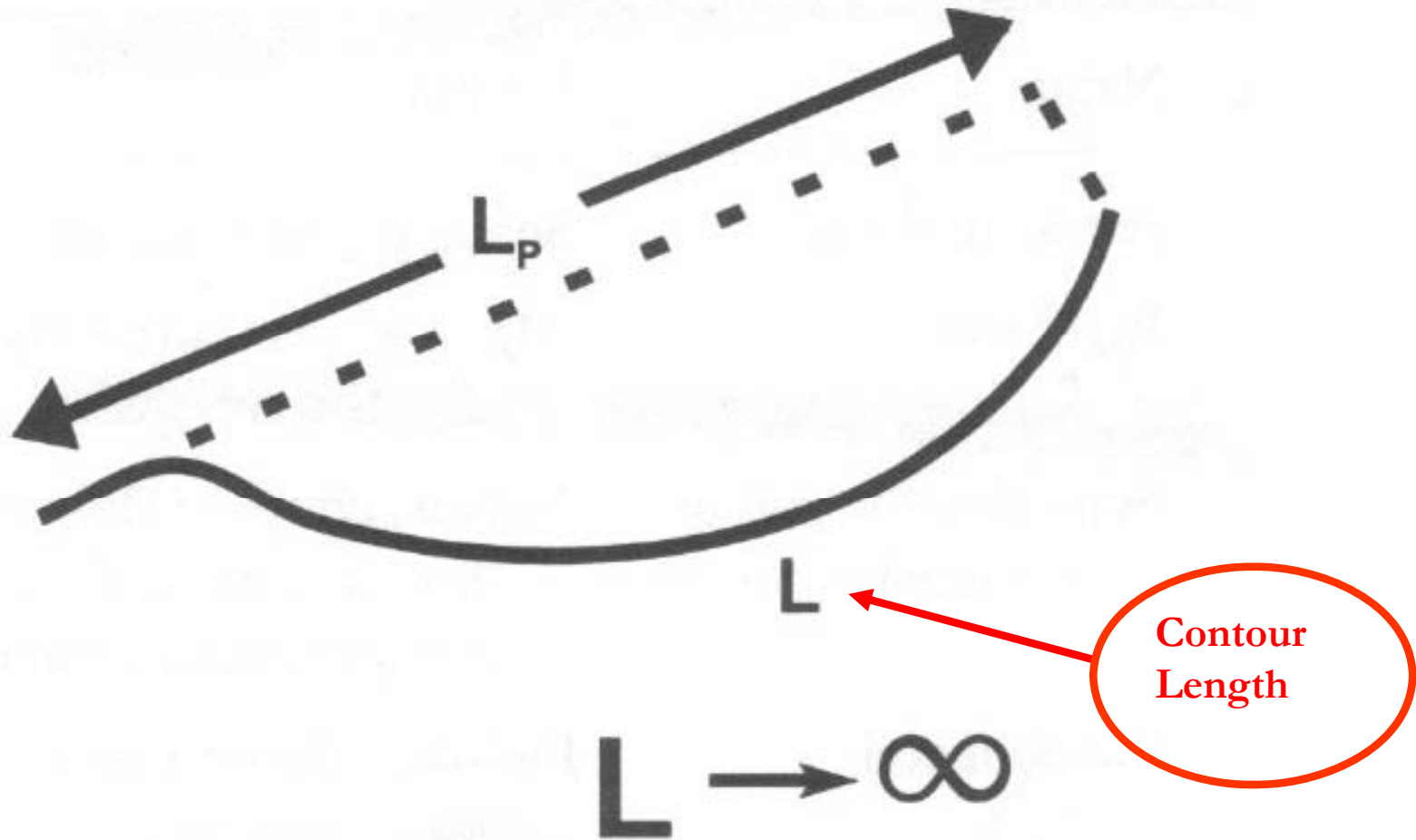


Pavlov, Rowe & Harding, *Trends in Analytical Chemistry*, 1997



Worm-like Chain

Flexibility parameter: Persistence length L_p



Kuhn-statistical length $\lambda^{-1} = 2L_p$

Worm-like Chain

Flexibility parameter: Persistence length L_p

Theoretical limits: Random coil $L_p = 0$
Rigid rod $L_p = \text{infinity}$

Practical limits: Random coil $L_p \sim 1\text{-}2\text{nm}$
Rigid rod $L_p \sim 200\text{nm}$

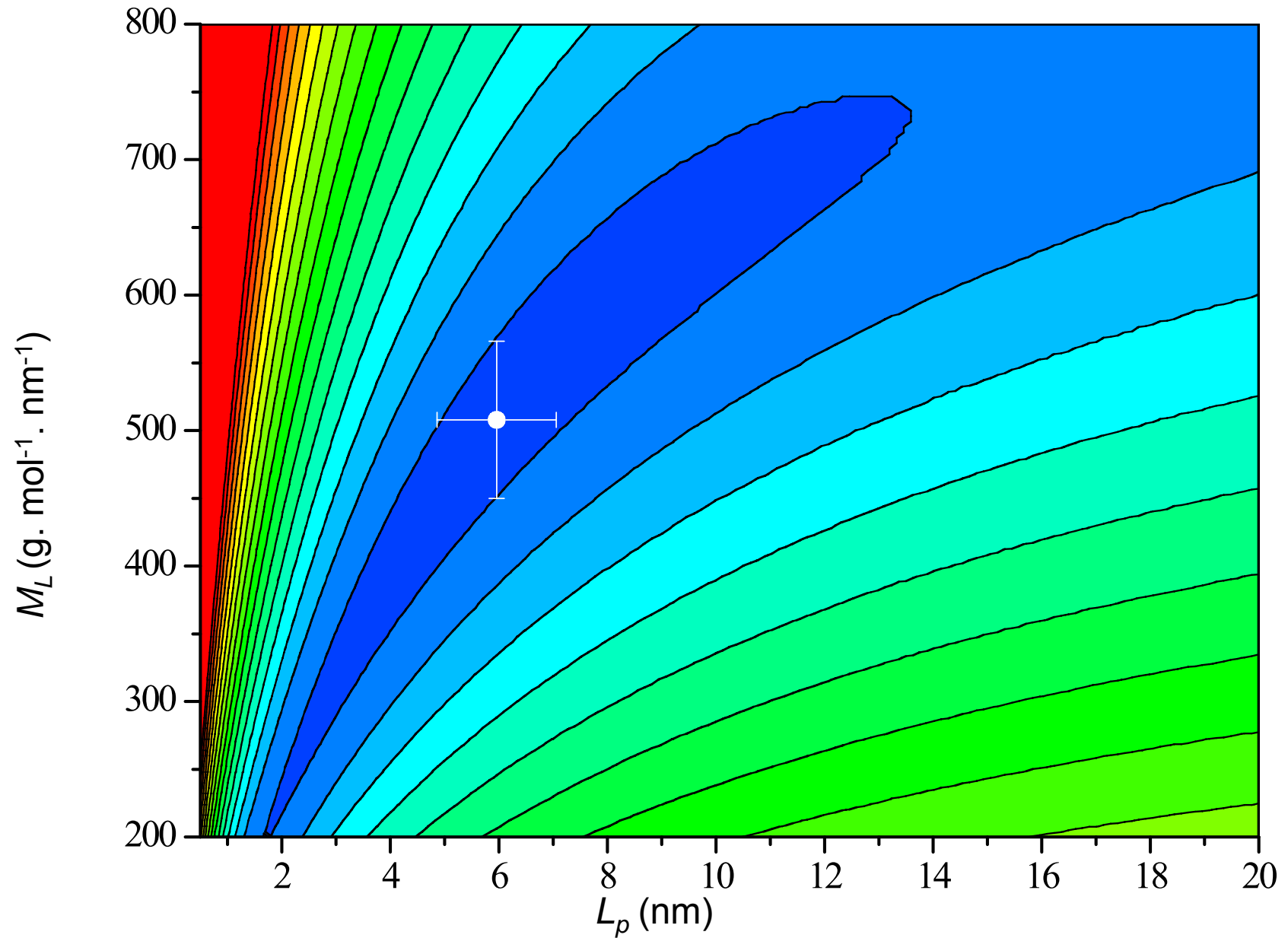
“Bushin-Bohdanecky” relation

$$\left(\frac{M_w^2}{[\eta]}\right)^{1/3} = A_0 M_L \Phi^{-1/3} + B_0 \Phi^{-1/3} \left(\frac{2L_p}{M_L}\right)^{-1/2} M_w^{1/2}$$

“Yamakawa-Fujii” relation

$$s^0 = \frac{M_L (1 - \bar{v} \rho_0)}{3\pi\eta_0 N_A} \times \left[1.843 \left(\frac{M_w}{2M_L L_p}\right)^{1/2} + A_2 + A_3 \left(\frac{M_w}{2M_L L_p}\right)^{-1/2} + \dots \right]$$

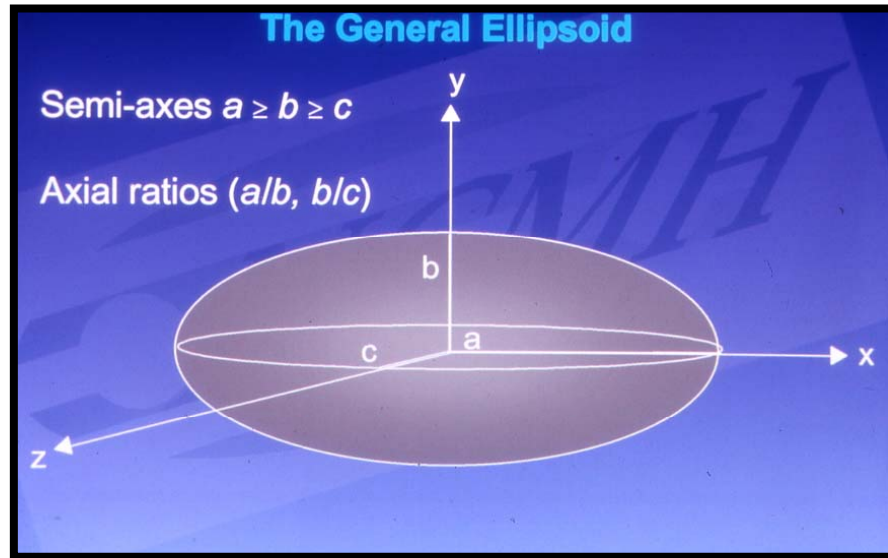
Global "Hydfit" plot: xyloglucan



Flexibilities of carbohydrate polymers

Carbohydrate Polymer	L_p (nm)
Pullulan	1.2-1.9
Amylose	2.8
Pectin (69% esterified)	12-15
Pectin (0% esterified)	34
DNA	45
Schizophyllan	115-200
Scleroglucan	180 \pm 30
Xanthan	210

Protein conformation: ellipsoids and beads

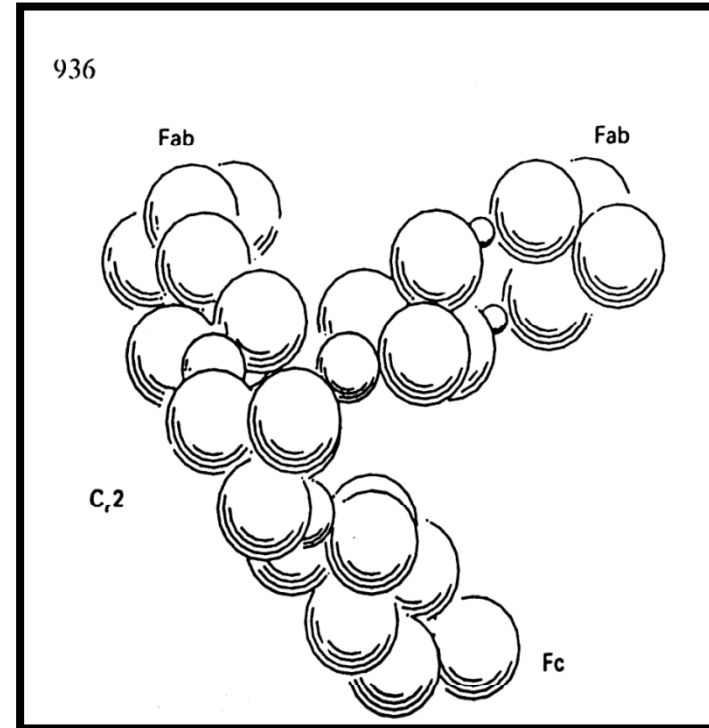


Software

<http://www.nottingham.ac.uk/ncmh>

Ellips1 (ellipsoids of revolution)

Ellips2, Ellips3, Ellips4 (general ellipsoids)



Software

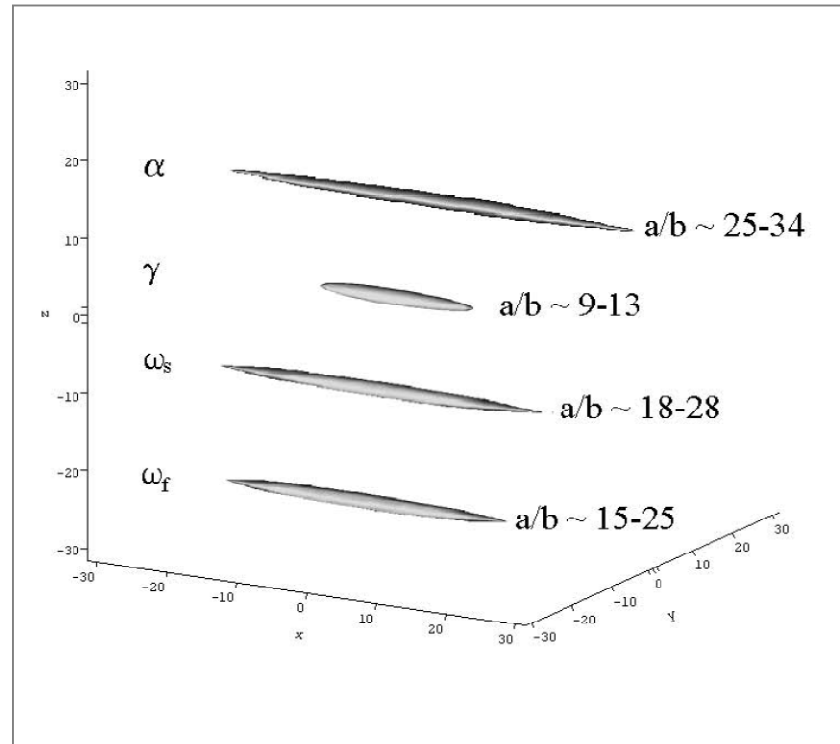
<http://leonardo.inf.um.es/macromol>

Hydro,

Solpro,

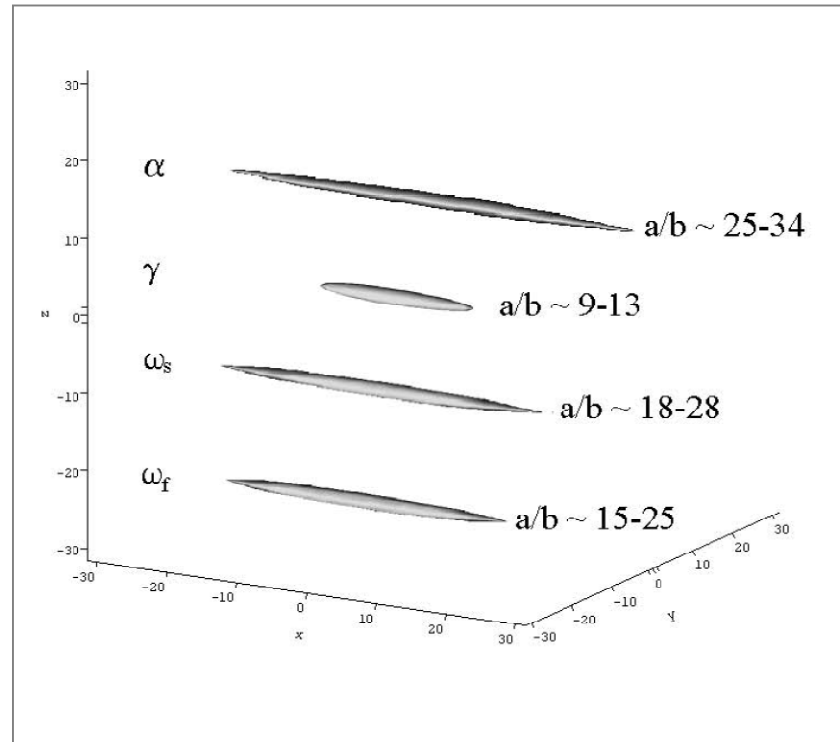
HydroPro

Ellipsoid axial ratio determinations – wheat protein gliadins



**Structure and heterogeneity of
gliadin: a hydrodynamic evaluation**
S. Ang et al, *Eur. Biophys. J.* (2009)

Ellipsoid axial ratio determinations – wheat protein gliadins



**Structure and heterogeneity of
gliadin: a hydrodynamic evaluation**
S. Ang et al, *Eur. Biophys. J.* (2009)

ELLIPS1

www.nottingham.ac.uk/ncmh

*Demonstration of ELLIPS1 & ELLIPS2 programs:
download from <http://www.nottingham.ac.uk/ncmh>*

For a wide variety of hydrodynamic parameters including v (from intrinsic viscosity)– see Lecture 1 notes or P (from sedimentation or diffusion measurements) – see Lecture 3 notes:

$$v = [\eta] / v_s$$

$$P = (f/f_o) \cdot (v/v_s)^{1/3}$$

$$\begin{aligned} \text{where } (f/f_o) &= (k_B T / 6\pi\eta_o) \{ (4\pi N_A / 3\bar{v}M)^{1/3} \} / D_{20,w}^o \\ &= (M(1-\bar{v}\rho_o) / N_A 6\pi\eta_o) \{ (4\pi N_A / 3\bar{v}M)^{1/3} \} / s_{20,w}^o \end{aligned}$$

$$\frac{f}{f_0} = \frac{k_B T}{6\pi\eta_0} \left(\frac{4\pi N_A}{3\bar{v}M} \right)^{1/3} \cdot \frac{1}{\eta_{20,W}^0}$$

$$= \frac{M(1-\bar{v}\rho)}{N_A 6\pi\eta_0} \left(\frac{4\pi N_A}{3\bar{v}M} \right)^{1/3} \cdot \frac{1}{\eta_{20,W}^0}$$

$$\rho = \left(\frac{f}{f_0} \right) \cdot \left(\frac{\bar{v}}{v_s} \right)^{1/3}$$

$$v_s = \text{swollen specific volume (ml/g)} = \bar{v} + \frac{\delta}{\rho_0}$$

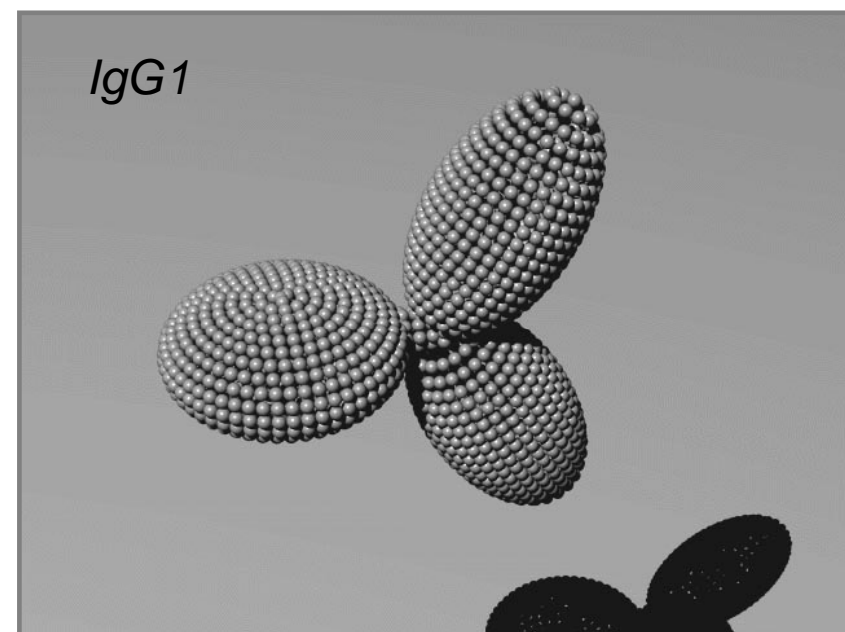
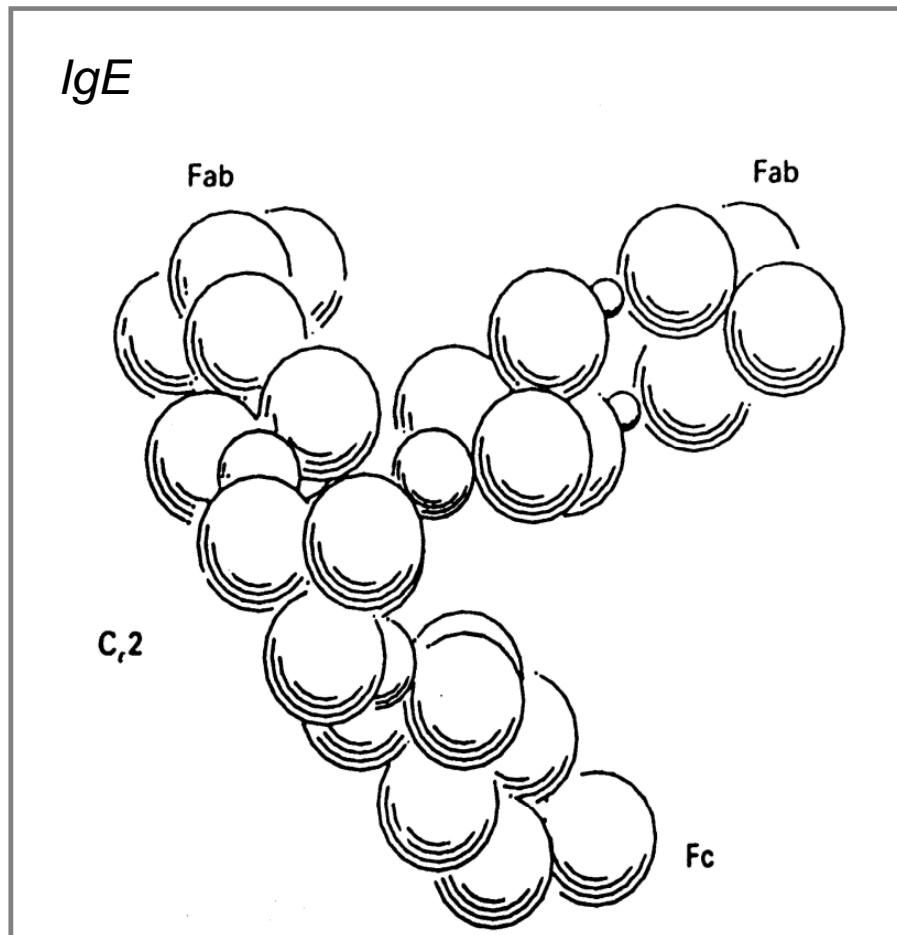
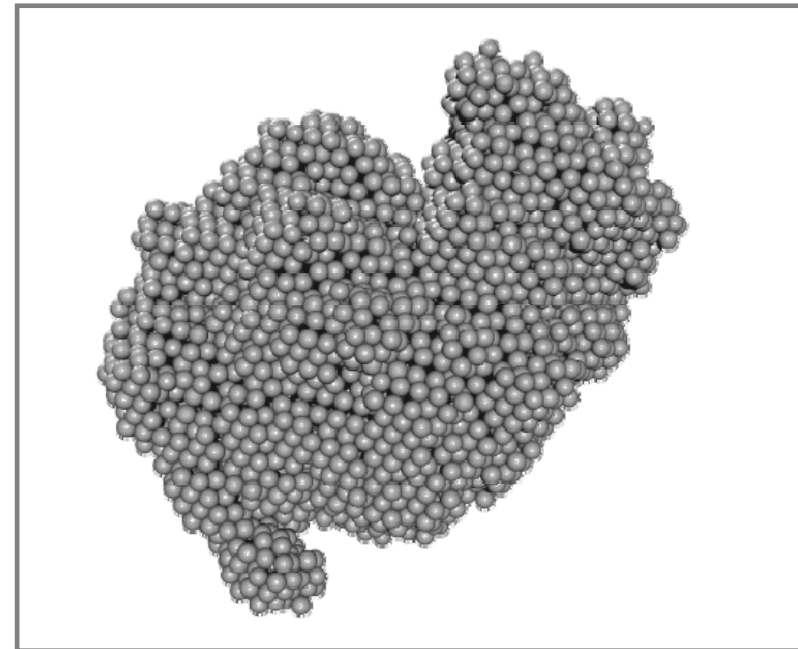
hydration

For more complicated shapes:

BEAD & SHELL MODELS

Hydro, Solpro, HydroPro etc

<http://leonardo.inf.um.es/macromol/>



Follow up bibliography:

1. Serydyuk, I.N., Zaccai, N.R. and Zaccai, J. (2006) *Methods in Molecular Biophysics*, Cambridge, Chapters D1 and D4
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